

On The Injection Spectrum of Ultrahigh Energy Cosmic Rays in the Top-Down Scenario

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Abstract

We analyze the uncertainties involved in obtaining the injection spectra of UHECR particles in the top-down scenario of their origin. We show that the DGLAP Q^2 evolution of fragmentation functions (FF) to $Q = M_X$ (mass of the X particle) from their initial values at low Q is subject to considerable uncertainties. We therefore argue that, for $x \lesssim 0.1$ (the x region of interest for most large M_X values of interest, $x \equiv 2E/M_X$ being the scaled energy variable), the FF obtained from DGLAP evolution is no more reliable than that provided, for example, by a simple Gaussian form (in the variable $\ln(1/x)$) obtained under the coherent branching approach to parton shower development process to lowest order in perturbative QCD. Additionally, we find that for $x \gtrsim 0.1$, the evolution in Q^2 of the singlet FF, which determines the injection spectrum, is “minimal” — the singlet FF changes by barely a factor of 2 after evolving it over ~ 14 orders of magnitude in $Q \sim M_X$. We, therefore, argue that as long as the measurement of the UHECR spectrum above $\sim 10^{20}$ eV is going to remain uncertain by a factor of 2 or larger, it is good enough for most practical purposes to directly use any one of the available initial parametrisations of the FFs in the x region $x \gtrsim 0.1$ based on low energy data, without evolving them to the requisite Q^2 value.

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1 Introduction

One of the main problems in understanding the origin of the observed Ultra-High Energy Cosmic Ray (UHECR) events with energy $E \gtrsim 10^{20}$ eV[1] — below we will sometimes refer to these as Extreme Energy Cosmic Ray (EECR) events — is the difficulty of producing such enormously energetic particles in astrophysical environments by means of known acceleration mechanisms. There are but a few astrophysical objects — among which are, perhaps, Gamma Ray Burst (GRB) sources and a class of powerful radio galaxies — where protons can in principle be accelerated to requisite energies (at source) of $\gtrsim 10^{21}$ eV by the standard diffusive shock acceleration mechanism albeit with optimistic assumptions on the values of the relevant parameters. However, even for these objects, their locations and spatial distributions are not easy to reconcile with the observed spectrum and large-scale isotropy of the UHECR particles. (For recent reviews on astrophysical source origin of EECR see, for example, Refs. [2, 3]).

An alternative mechanism of producing the EECR particles is provided by the so-called “top-down” (TD) scenario (see [4] for a review) in which the EECR particles are envisaged to result from *decay* of some sufficiently massive particles, generically called “X” particles, of mass $M_X \gg 10^{20}$ eV, which could originate from processes in the early Universe. This is in contrast to the conventional “bottom-up” scenario in which *all* cosmic ray particles including the EECRs are thought to be produced through processes that accelerate particles from low energies to the requisite high energies in suitable astrophysical environments.

The X particles of the TD scenario, if at all they exist in Nature, are most likely to be associated with some kind of new physics at some sufficiently high energy scale that could have been realized in an appropriately early stage of the Universe. Two possibilities for the origin of the X particles have been discussed in the literature: They could be short-lived particles released in the Universe today from cosmic topological defects such as cosmic strings, magnetic monopoles, etc. [5] formed in a symmetry-breaking phase transition in the early Universe. Alternatively, they could be some metastable (and currently decaying) particle species with lifetime larger than or of the order of the age of the Universe.

Since the mass scale M_X of the hypothesized X particle is well above the energy scale currently available in accelerators, its primary decay modes are unknown and likely to involve elementary particles and interactions that belong to unknown physics beyond the Standard Model (SM). However, irrespective of the primary decay products of the X particle, the observed UHECR particles must eventually result largely from “fragmentation” of the Standard Model quarks and gluons, that come from the primary

decay products of the X particles, into hadrons. The most abundant final observable particle species in the TD scenario are expected to be photons and neutrinos from the decay of the neutral and charged pions, respectively, created in the parton fragmentation process, together with a few percent baryons (nucleons). The injection- or the source spectra of various species of UHECR particles (nucleons, photons and neutrinos) in this TD scenario are thus ultimately determined by the physics of the parton fragmentation process. The final observable UHECR particle spectra are determined by further processing of these injection spectra due to extragalactic and/or Galactic propagation effects depending on where the X particle decay takes place. Clearly, in order to test the predictions of the TD scenario against UHECR experimental data, it is crucial to be able to reliably calculate the injection spectra of various UHECR particles in this scenario. This is the subject we concern ourselves with in this paper.

The problem at hand is essentially the same as determining the single-particle inclusive spectrum of hadrons produced, for instance, in the process $e^+e^- \rightarrow \gamma/Z \rightarrow q\bar{q} \rightarrow$ hadrons (see, for example, [6]). The primary quarks produced in the collision would in general not be on-shell and would have large time-like virtuality $Q \sim \sqrt{s}$, the center-of-mass energy of the process. Each quark would, therefore, reduce its virtuality by radiating a gluon, the latter in turn splitting into a $q\bar{q}$ pair or into two gluons, and so on. This process gives rise to a parton shower whereby at each stage a virtual parton splits into two other partons of reduced virtualities. This process of parton shower development is well-described by perturbative QCD until the virtuality reduces to $Q = Q_{\text{hadron}} \sim 1 \text{ GeV}$ when non-perturbative effects come into play binding partons into colorless hadrons. In the end, the link between partons and hadrons is quantitatively described in terms of fragmentation functions (FFs) $D_a^h(x, Q)$, which give the probability that a parton a produced with an initial virtuality $Q = \sqrt{s}$ produces the hadron h carrying a fraction $x \equiv 2E/\sqrt{s}$ of the energy of a (E being the energy of the hadron)³. The final single particle inclusive spectrum of hadrons is given by a convolution of these FFs with the production probabilities of the primary partons (see next section).

In the same way, the problem of determining the injection spectrum of UHECR particles from the decay of X particles essentially reduces to determining the FFs $D_a^h(x, M_X)$ for various hadron species h (pions, nucleons) where a represents the primary partons to which the X particle decays. (Actually, in our present case, we will be interested only in the so-called “singlet” FF corresponding to a sum over all partons a as explained later).

Clearly, the FFs themselves cannot be directly calculated from first principles en-

³At high energies E of our interest throughout this paper we shall assume $E \simeq p$, the momentum of the particle.

tirely within perturbative QCD without extra assumptions about the nature of the non-perturbative process of formation of hadrons from partons. Several different approaches have been taken in the recent literature for evaluating the relevant FFs, which are discussed below.

In this paper, we critically examine one of the approaches of evaluating the relevant FFs, namely, the DGLAP evolution equation method [7, 8, 9, 10, 11], that has been widely used in recent calculations of the UHECR injection spectra in the TD scenario. We discuss the inherent uncertainties involved in this approach in calculating the relevant FFs over the ranges of x and M_X of interest. We also compare the FFs so obtained with those given by a simple analytical expression (given by a Gaussian in the variable $\ln(1/x)$ as discussed later) obtained within the context of an analytical approach, namely, the coherent branching formalism, to lowest order in perturbative QCD [6], this analytical approach being valid only under “small” x and “large” Q approximation. We show that except for “large” $x \gtrsim 0.1$, the uncertainties involved in obtaining the relevant FFs by numerical solution of the DGLAP evolution equation do not allow much significant advantage of using this numerical method over the simple analytical (but approximate) formula for FFs provided by the coherent branching approach. At the same time, we also find that, in the region $x \gtrsim 0.1$, the evolution (in Q) of the *singlet* FFs (which is what we are interested in) is very little — the singlet FF changes by only a factor of 2 or so after evolving it over ~ 14 orders of magnitude in $Q \sim M_X$. We explain the reason for this, and argue that, as long as the measurement of the EECR spectrum is going to remain uncertain within a factor of 2 or larger (which is likely to be the case in the foreseeable future), it is good enough for most practical purposes to directly use any one of the available parametrisations of the FFs in the x region $x \gtrsim 0.1$ based on low energy (say at the Z-pole) data from $e^+e^- \rightarrow$ hadrons experiments even without evolving them in Q by means of DGLAP evolution equation.

As mentioned above, the X particle decay process may involve particles and interactions belonging to possible new physics beyond SM. Most of the recent studies using DGLAP evolution equation method have been done in the context of a particular model of the possible new physics beyond SM, namely, the Minimal Supersymmetric Standard Model (MSSM). While these studies are certainly useful, there exists, however, no direct evidence yet of Supersymmetry in general and the MSSM in particular. Indeed, the unknown nature of the physics beyond SM introduces additional uncertainties in the whole problem over and above the intrinsic uncertainties associated with the DGLAP evolution method itself which is fundamentally based on standard QCD. In order to analyze these uncertainties associated with the DGLAP evolution method itself, we restrict our analysis here to the standard DGLAP evolution equa-

tions for FFs based on QCD. Also, to keep our analysis simple, we shall illustrate our main results by considering the behavior of the FF for only one of the hadron species, namely, pions; our general conclusion, however, apply to nucleons as well as to other mesons like the K meson, too.

The rest of this paper is organized as follows: In the following section we set our notations and express the energy spectrum of hadrons resulting from the decay of the X particle in terms of the singlet fragmentation function (FF). In section 3, we review the various methods of evaluating the FF. Our main results are presented and discussed in section 4, and brief conclusions are presented in section 5.

2 Fragmentation Functions

Let us consider the situation when the X decays from rest into a q and a \bar{q} pair (where q can be u, d, s, c, b, t) which subsequently hadronize: $X \rightarrow q\bar{q} \rightarrow h + \dots$ (here h is a hadron). This is to facilitate direct comparison (at low c. m. energies of $\sqrt{s} \sim 100$ GeV) with the available data on the similar process $e^+e^- \rightarrow \gamma/Z \rightarrow q\bar{q} \rightarrow h + \dots$. We are interested in the energy spectrum or the single-particle energy distribution of the hadron species h , dN^h/dx , where $x \equiv 2E_h/M_X \leq 1$ is the scaled hadron energy. This can be written as a sum of contributions from different primary quarks $a = u, d, \dots$ (and their antiparticles) as [6]

$$\frac{dN^h}{dx}(x, s) \propto \sum_a \int_x^1 \frac{dz}{z} \frac{d\Gamma_{X \rightarrow a}}{dz}(z, s) D_a^h(x/z, s), \quad (1)$$

where $d\Gamma_{X \rightarrow a}/dz$, the decay width of the X into parton a , is calculable in perturbation theory, and D_a^h is the perturbatively non-calculable parton-to-hadron fragmentation function (FF).

Since the mass scale M_X is much larger than the electroweak scale, we shall assume, following earlier work [8], flavor universality in the decay of X, which means that all primary quark flavors are produced with equal probability. This, together with the fact that, to lowest order for a 2-body decay, $d\Gamma_{X \rightarrow a}/dz \propto \delta(1 - z)$, gives

$$\frac{dN^h}{dx}(x, s) \propto \sum_a D_a^h(x, s) \equiv D_S^h, \quad (2)$$

where D_S^h is the *singlet* FF [6].

The proportionality constant of equation (2) can be determined from the energy

conservation condition for hadronization of each individual quark, namely,

$$\sum_h \int_0^1 dx x D_a^h(x, s) = 1, \quad (3)$$

together with the condition for overall energy conservation in the entire hadronization process, i.e.,

$$\sum_h \int_0^1 dx x \frac{dN^h}{dx}(x, s) = 2. \quad (4)$$

This finally gives

$$\frac{dN^h}{dx}(x, s) = \frac{1}{n_F} D_S^h(x, s), \quad (5)$$

where n_F is the number of active quark flavors.

3 Evaluation of FFs

Three approaches to the problem of evaluating the relevant FFs have been followed in the literature. Below we discuss these in turn:

3.1 Using DGLAP evolution equation for FFs

Although the FFs themselves are not directly calculable entirely within perturbative QCD, given their x dependence extracted from experimental data at some scale Q_0^2 , the *evolution* of the FFs with Q^2 is computable within perturbative QCD, and is given by the DGLAP evolution equation for FFs [6]. The relevant FFs at the scale $Q = M_X$ can then be evaluated by numerically solving the DGLAP evolution equation for the FFs, starting with input FFs extracted from e^+e^- data at some laboratory energy scale, e.g., on the Z-pole ($Q_0 = 91$ GeV). This method has been used, for example, in Refs. [8, 9, 10, 11] to obtain the injection spectra of UHECR particles in the TD scenario.

3.1.1 Numerical solution of DGLAP evolution equation for FFs

The DGLAP evolution equation for the FF is given by a form similar to that for parton distribution functions [6]

$$t \frac{\partial}{\partial t} D_i(x, t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ji}(z, \alpha_s) D_j(x/z, t), \quad (6)$$

where the symbols have their usual meaning [6] and, as is well-known, the splitting function is P_{ji} instead of P_{ij} . These splitting functions have perturbative expansions

in powers of the strong coupling α_s and we have taken the Lowest Order (LO) expressions for these in our calculations of the LO DGLAP evolution for the FF. In practice, one considers non singlet fragmentation combinations (in flavor space) of the form $D_{NS} = D_{q_i} - D_{q_j}$ (where i, j run over both quark and anti-quark flavors) so that the flavor singlet gluons drop out, and the singlet combinations $D_S = \sum_i D_{q_i}$ which mixes with the fragmentation of the gluon, giving a matrix relation. Due to the $1/x$ pole in the P_{gg} splitting function, the sea contribution increases significantly at low x for larger Q^2 . In fact, the effect of splitting is the same for distribution and fragmentation functions — as the scale of evolution Q^2 increases, the x distribution is shifted towards lower values.

The evolution equations are usually solved numerically in Mellin space. However, for convenience, we have used a numerical solution of these equations in real space.

There are various parametrisations for FFs available in the literature, given, for example, by KKP [12], BKK [13], and by Kretzer [14], the most recent being those of KKP and Kretzer. These provide simple parametrisations of the FFs as functions of x and Q^2 that are intended to reproduce their evolved values (obtained by solving the time-like evolution equations) within the range of validity of their parametrisations. Most of these parametrisations do not work below around $x \simeq 0.05$ or at the ultra high energy values of Q^2 that we are ultimately interested in.

Therefore, for numerical accuracy, we have not used any of the *parametrisations* provided by these groups. We have taken the x distributions of these FFs at their starting scale Q_0^2 and evolved them through the DGLAP equations to higher values of Q^2 . This allows us to reach much higher values of Q^2 and very low values of $x \lesssim 10^{-8}$ as is required for our analysis, way beyond the range of validity of the simple *parametrisations* provided. It is, of course, not clear whether even these starting values are reliable over such enormous ranges of x and Q^2 . For the present, however, we will assume that these starting parametrisations are reliable as long as we do not reach ultra low values of x where the phenomenon of coherent branching makes the FFs turn downwards as we go to lower x (see below).

In what follows, when we talk of a particular parametrisation (KKP, BKK or Kretzer) it should be understood to mean that we use the initial parametrisations provided by these groups and evolve them through the evolution equations, and do not use the algebraic parametrisations given by the authors valid over a restricted range of Q^2 and x .

3.2 Monte Carlo simulation

In this approach one performs a direct numerical simulation of the parton shower process described by perturbative QCD coupled with a numerical modeling of the non-perturbative hadronization process. In the context of TD scenario of UHECR origin this “Monte Carlo” (MC) method has been studied in Refs. [15, 11]. A comparison of the DGLAP evolution and MC methods of obtaining the relevant FFs has recently been done in [11].

3.3 Coherent Branching, Modified Leading-Log Approximation and Local Parton-Hadron Duality: An analytical approach

This is essentially an analytical approach entirely within perturbative QCD in which the parton-to-hadron singlet FFs are obtained from an analytical solution, obtained *under large \sqrt{s} and small x approximations*, of a modified form of the DGLAP evolution equation that describes the parton shower evolution process within the so-called “coherent branching” formalism [6]. The method assumes perturbative QCD to be valid all the way down to a virtuality of $\sim \Lambda_{\text{eff}}$, an “effective” QCD scale of order few hundred MeV, and essentially gives the perturbative gluon-to-gluon fragmentation function which dominates all FFs at small x . The FFs to different hadrons are taken to be proportional to this gluon-to-gluon FF with appropriate normalization constants determined from $e^+e^- \rightarrow$ hadrons data in accordance with the hypothesis of Local Parton Hadron Duality [16] which, at a purely phenomenological level, seems to describe the experimental data rather well [6]. Although there is no “proof” of the LPHD hypothesis at a fundamental theoretical level yet, the basis of the LPHD hypothesis is that the actual hadronization process occurs at a low virtuality scale of order of a typical hadron mass independent of the energy of the cascade initiating primary parton, and involves only low momentum transfers and local color re-arrangement which do not drastically alter the form of the momentum spectrum of the particles in the parton cascade already determined by the “hard” (i.e., large momentum transfer) perturbative QCD processes. Thus, the non-perturbative hadronization effects are lumped together in an “unimportant” overall normalization constant which can be determined phenomenologically.

The modification of the DGLAP evolution equation referred to above consists of ordering the basic parton splitting processes (that give rise to parton shower development) according to decreasing emission angles between the final-state partons rather than their decreasing virtuality. This angular ordering is due to the color coherence phenomenon which leads to suppression of soft gluon emission, making the FFs

turnover at small x below a characteristic value $x_c \sim (0.1 \text{ GeV}/\sqrt{s})^{1/2}$ — an effect clearly seen in the experimental data [17].

To leading order, the solution of the above mentioned modified DGLAP equation gives the following Gaussian form for the singlet FF, D_S (dropping the superscript h), in the variable $\xi \equiv \ln(1/x)$ [6]:

$$D_S(\xi) \equiv xD_S(x, s) \propto \exp \left[-\frac{1}{2\sigma^2} (\xi - \xi_p)^2 \right], \quad (7)$$

where the peak position $\xi_p = Y/2$, and $2\sigma^2 = (bY^3/36N_c)^{1/2}$, with $Y \equiv \ln(Q/\Lambda_{\text{eff}}) = \ln(m_X/\Lambda_{\text{eff}})$ and $b = (11N_c - 2n_F)/3$, $N_c = 3$ being the number of colors.

Including the next-to-leading order corrections, calculated in an analytical framework known as Modified Leading-Log Approximation (MLLA)[18], yields again a closed form analytical expression for FFs that, as functions of the variable ξ , can be well approximated by a “distorted Gaussian” [18] in terms of calculable higher moments of the variable ξ . The above Gaussian expression is a good approximation to the full MLLA result for ξ not too far away on either side from the peak position ξ_p . The peak position ξ_p also defines for us what we mean by “small” x approximation: The MLLA (and its Gaussian approximation) are expected to be valid for x not too large compared to $x_c \simeq (\Lambda_{\text{eff}}/Q)^{1/2}$.

Within the LPHD picture, there is no way of distinguishing between various different species of hadrons, all of which would thus have the same spectral shape. Phenomenologically, the experimental data at laboratory energies can be fitted by using different values of Λ_{eff} for different species of particles depending on their masses. For our consideration of particles at EECR energies, however, all particles are extremely relativistic (and hence essentially massless), and all hadron species have essentially the same spectral *shape* which, will be relatively insensitive to the exact value of Λ_{eff} since $\sqrt{s} \sim M_X \gg \Lambda_{\text{eff}}$.

Below, we shall compare the singlet FF obtained within the coherent branching formalism described above with that obtained from numerical solution of the DGLAP evolution equation. Since we consider DGLAP evolution for the singlet FF only to leading order (LO), to be consistent, and for simplicity, we shall use the corresponding LO result, namely, the Gaussian expression given by eq. (7) instead of the full MLLA result. The Gaussian approximation (which we shall refer to as “MLLA-Gaussian” hereafter) becomes an increasingly better approximation to the full MLLA result at increasingly higher \sqrt{s} .

An important point to note here is that, at laboratory energies, MLLA gives a very good fit to the data at essentially “*all*” x values (including “large” x) for which data

exist [18], although the MLLA analytic result is based on small x approximation. For example, for $\Lambda_{\text{eff}} = 200 \text{ MeV}$ (which value we shall assume throughout this paper for illustration of the relevant numbers) and $\sqrt{s} = 91 \text{ GeV}$, we have $x_c \simeq 0.05$. However, as shown in Figure 1 below, the simple Gaussian curve provides a very good fit to the 91 GeV data at least up to $x \simeq 0.3$ and reasonably good fit at even larger values of x . Since the width of the Gaussian, σ , increases with \sqrt{s} (albeit only logarithmically), we may expect the MLLA (Gaussian) to provide, with increasing \sqrt{s} , increasingly better description of reality at increasingly larger values of x beyond the corresponding x_c values.

Actually, this fact — that MLLA results provide good description of the data even at relatively “large” x although it was derived under small x approximation — was already noticed in [19, 18] where this agreement was termed as “natural, though accidental”. The technical reason for this “coincidence” was also explained there; we shall, however, not go into these technical aspects in this paper.

4 Results and Discussions

As a test of our DGLAP evolution code we show in Figure 1a the comparison of the results of DGLAP evolution of the singlet FF for pion ($\pi^+ + \pi^-$) with experimental data at 91.2 GeV [20] for the three different initial parametrisation (KKP, BKK, Kretzer) of the FFs. And Figure 1b shows the corresponding $D(\xi)$ vs ξ curves.

The calculations are in overall good agreement with the data, as expected. For comparison, we also display the MLLA-Gaussian curve. As mentioned in the last section, the MLLA-Gaussian fits the data at large x reasonably well. In fact, the Gaussian provides a better description of the data than the DGLAP results even at moderately large $x \sim 0.5$. And, as expected, at small x ($x \lesssim 0.1$) (i.e., $\xi \gtrsim 2.3$), the DGLAP results fail rather badly whereas the Gaussian gives an excellent fit. The reason for this is clear: The phenomenon of coherent branching dominates the parton shower process at low x . The standard DGLAP evolution equation for FF does not take this phenomenon into account, and the resulting FFs obtained from numerical solution of the DGLAP evolution equation are, therefore, not expected to be valid for $x \lesssim x_c \sim 0.05$ (for $\sqrt{s} = 91.2 \text{ GeV}$). (Actually, as seen from the figures, the DGLAP already fails at an x value somewhat larger than this value of x_c).

In Figure 2 we show the results for the singlet $D(x)$ (for pions) at various values of M_X up to $M_X = 10^{16} \text{ GeV}$ obtained by solving the DGLAP equation for three different initial FF parametrisations. Again, for comparison we also show the MLLA-Gaussian curves.

In Figure 3 we show the $D(\xi)$ vs $\xi = \ln(1/x)$ curves for the same set of parametrisations as in Figure 2.

In Figures 2 and 3, we have normalized the Gaussian curves with the DGLAP evolution results at $x \simeq 0.03$ where the results of all three FF parametrisations agree. It can be seen from Figures 2 and 3 that there are large discrepancies amongst the results of the three different initial parametrisations for $x \lesssim 10^{-2}$. Note that these discrepancies are at x regions well above the turning points of the FFs that are due to coherence effects, and are therefore to be attributed to magnification (due to Q^2 evolution) of the intrinsic differences amongst the three initial parametrisations. The parametrisations are done by fitting the FFs to the known data which go only up to $\sqrt{s} \sim 190$ GeV. Moreover, most parametrisations (including KKP) are restricted to x region above ~ 0.05 (because there are no data for lower x at the initial scale of parametrisation). So the resulting initial parametrisations do not satisfy the various sum rules very well. For example, the momentum (or energy) sum rule is rather poorly satisfied in KKP. Also, the behavior of $D(x, Q)$ shows some strange behavior as illustrated more clearly in Figures 4 a–c where we show the behavior of FF as a function of x for different values of Q for KKP, BKK and Kretzer parametrisations.

On standard theoretical ground, it is expected that with increasing Q , the x distribution should shift towards lower values, i.e., the FF should increase with Q at low x and decrease at large x . In effect, this implies a steepening of the particle spectrum with increasing Q . Thus, the FFs as a function of x for two different values of Q should cross at some x . However, the curves in Figures 4a–c do not show this expected crossing behavior except marginally for the BKK parametrisation (Figure 4a) at low Q values (specifically the $Q = 10$ and 90 GeV curves). This is a reflection of the fact that the data available at existing energies (on which the parametrisations are based) show this behavior clearly only at low Q ($\sqrt{s} < 50$ GeV), while being essentially flat beyond this value for all x (see, e.g., Figure 15.1(b) in Ref. [17]). Moreover, none of the parametrisations use the low x data which do show slight increase with Q (see Figure 15.1(b) in Ref. [17]). Consequently, our evolution results based on these parametrisations also do not show this effect. In fact, the $Q = 10^{16}$ GeV curve is always substantially below the curves for lower Q for all x reflecting the above facts.

The above results illustrate the fact that using DGLAP evolution to predict the shape of the UHECR injection spectra is subject to considerable uncertainty associated with the initial FF parametrisations.

The other important point to notice is that the effect of the evolution of the singlet FF with Q^2 is “minimal”. In fact, over the whole range of M_X from $91 - 10^{16}$ GeV, the FF changes only by a factor ~ 2 (see Figure 2). The reasons for this is that

the Q^2 evolution of the FFs is driven mainly by the gluon. However, in our case, particularly at very large Q and small x , the gluon FF is several orders of magnitude smaller than the singlet FF. Therefore, the evolution of the gluon FF has very little effect on the singlet FF. Actually, with the initial parametrisations used here, the singlet is 4 orders of magnitude larger than the gluon even at smaller Q (2.5 GeV) for small x ($\sim 10^{-7}$). Hence over the whole range in Q (i.e., up to $M_X \sim 10^{16}$ GeV), there is very little evolution with Q .

So it appears that full DGLAP evolution is essentially unnecessary at the current level of measurement of the UHECR spectra which are, and likely to remain in the foreseeable future, uncertain by factors larger than 2 or so. A typical parametrisation of the FFs is of the form $\sim x^\alpha(1-x)^\beta$ with α and β being functions of Q^2 . However, the above discussion seems to suggest that, as far as the singlet FF (for a given hadron species) is concerned, it is sufficient to obtain it directly from the individual FFs of different partons as given by the above form with appropriate values of the parameters α and β extracted from the relevant experimental data at some laboratory energy scale Q_0 .

To what extent can one use the MLLA at all x values of interest, namely, in the region $x \lesssim 0.1$? While, as we have seen, the MLLA describes the data well essentially at all x for $Q = 91$ GeV, the situation becomes more complicated for larger values of $\sqrt{s} = M_X$. For $M_X = 10^{13}$ GeV, for example, the coherent branching effect becomes important only at “ultra-low” $x \lesssim x_c \sim 1.4 \times 10^{-7}$. At the same time, the TD scenario of UHECR origin is generally relevant only for observed UHECR energies $E > 10^{10}$ GeV, which corresponds to $x > 2 \times 10^{-3} \gg x_c$ for $M_X = 10^{13}$ GeV. Thus, the coherent branching effects are not yet “switched on”, and it is not a priori clear whether the MLLA expression for the FF is valid at such relatively “large” x . This is the basis of the argument that one should not use the MLLA results in these circumstances; instead, one should obtain the relevant FFs by solving the DGLAP evolution equation for FF. While this is perhaps what one *should* do, the problem here is that the starting parametrisations of the FFs are not known at such values of x , and one has to extrapolate the starting FFs well below the lowest x value (~ 0.05) up to which the initial parametrisations of the FFs are known. This extrapolation is fraught with considerable uncertainty since one has to *assume*, a priori, a form of the extrapolated FF, and, as discussed above, simple extrapolation of the existing FF parametrisations to small x values gives widely different answers when evolved to high M_X values by means of DGLAP evolution equation.

In Ref. [10], the guiding principles adopted for extrapolation of the starting FFs to the relevant low x values are energy conservation and continuity of the FFs. These conditions, however, do not uniquely fix the form of the FFs valid over the entire

range of x of interest. In addition, to impose energy conservation, Ref. [10] had to assume FFs for all hadrons to have the same power-law form at low x , based on MLLA-LPHD result. The interesting result of Ref. [10], however, is that the resulting FFs obtained by solving the DGLAP evolution equation at high M_X smoothly match onto the properly normalized MLLA result at an x value which is considerably larger than the corresponding value of x_c . This suggests that one might as well use the MLLA-LPHD formula in the region $x \lesssim 0.1$, considering the uncertainties involved in DGLAP evolution of FFs.

5 Summary and conclusions

In this paper we have analyzed the uncertainties involved in obtaining the injection spectra of UHECR particles in the top-down scenario of their origin. We have demonstrated that evaluating the relevant FFs at the values of M_X and x of interest by evolving them (in $Q = M_X$) from their initial (parametrised) values at low Q by numerically solving the DGLAP evolution equation for FF is subject to considerable uncertainties. Indeed, we find that for $x \lesssim 0.1$ (the x region of interest for most large values of M_X of interest), the FF obtained from DGLAP evolution cannot be said to be any more reliable than that provided by the simple Gaussian form (in the variable ξ) based on coherent branching approach to parton shower development. At the same time, we also find that for $x \gtrsim 0.1$, the evolution of the singlet FF, which determines the injection spectrum, is “minimal” — the singlet FF changes by barely a factor of 2 after evolving over ~ 14 orders of magnitude in $Q \sim M_X$. We, therefore, argue that as long as the measurement of the EECR spectrum is going to remain uncertain by a factor of 2 or larger (which is likely to be the case in the foreseeable future), it is good enough for most practical purposes to directly use any one of the available *initial* parametrisations of the FFs in the x region $x \gtrsim 0.1$ based on low energy (say at the Z-pole) data from $e^+e^- \rightarrow$ hadrons experiments, without any need for evolving them to the required EECR Q^2 value.

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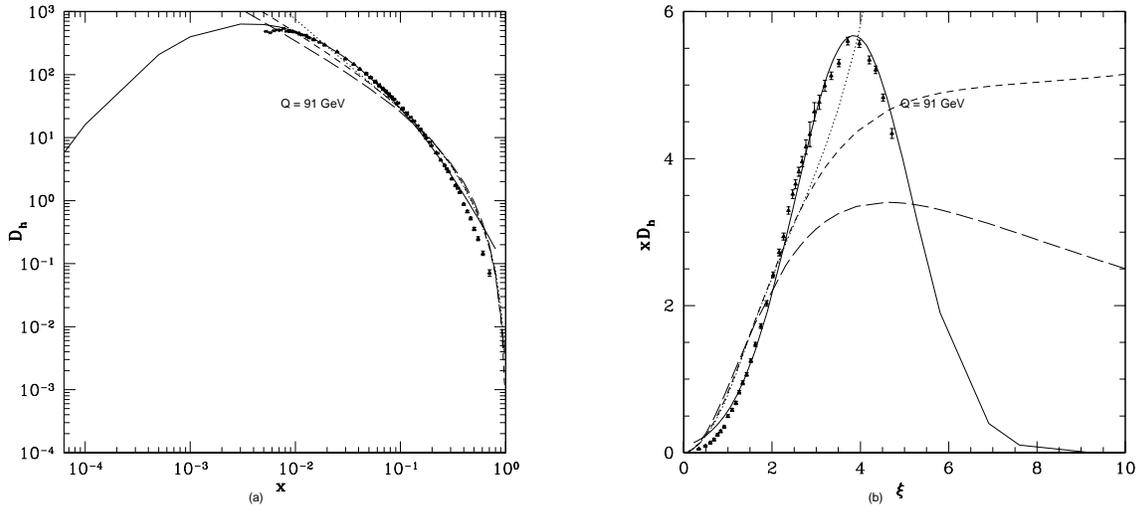


Figure 1: $D(x)$ and $D(\xi)$ curves along with 91 GeV data for three different parametrisations: KKP (dotted), BKK (short-dash) and Kretzer (long-dash). Also shown is the MLLA-Gaussian curve (solid line) given by equation (7) with normalization fixed by average pion multiplicity data.

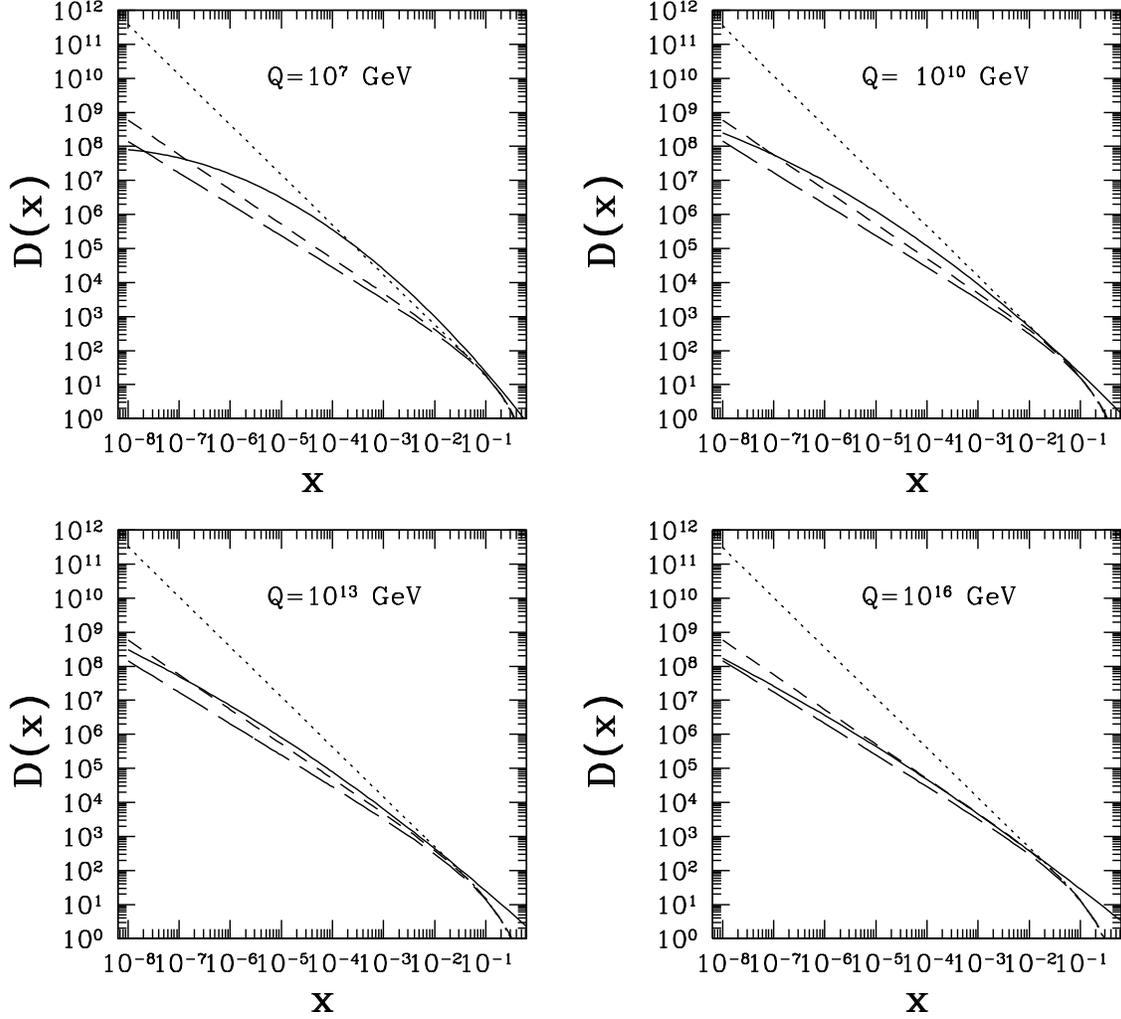


Figure 2: A comparison of $D(x)$ vs x curves at various different values of $Q = M_X$ for the three different FF parametrisations : KKP (dotted), BKK (short-dash) and Kretzer (long-dash). The solid curves represent the MLLA-Gaussian.

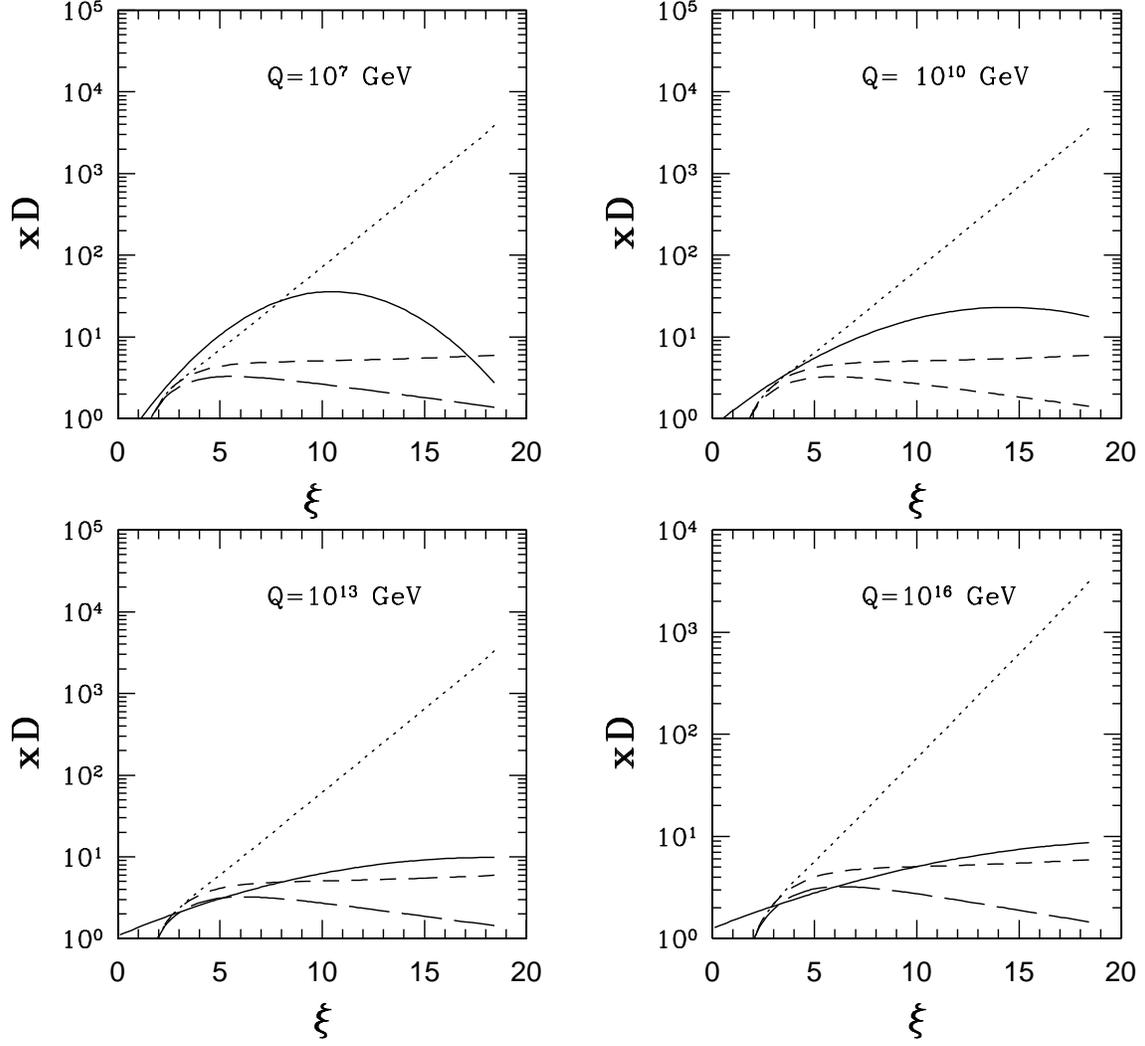


Figure 3: A comparison of $D(\xi)$ vs ξ curves at various different values of $Q = M_X$ for the three different FF parametrisations : KKP (dotted), BKK (short-dash) and Kretzer (long-dash). The solid curves represent the MLLA-Gaussian.

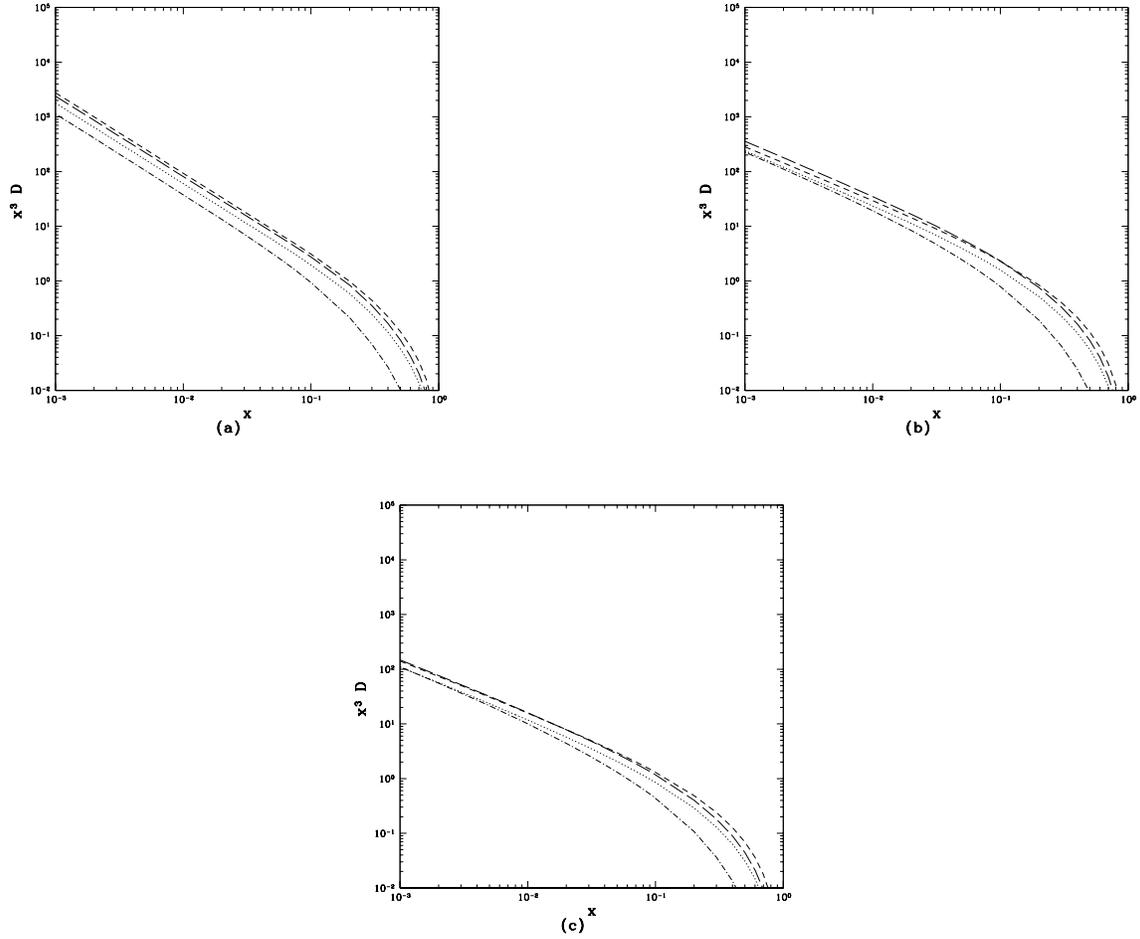


Figure 4: A plot of $x^3 D(x, Q)$ vs. x for (a) KKP (b) BKK and (c) Kretzer FF parametrizations. For each of these, the lines correspond to $Q=10$ (short-dash), 91 (long-dash), 189 (dot) and 10^{16} (dot-short dash) GeV.