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Escape velocities of interacting spherical galaxies

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Abstract. The dependence of the escape velocity of a pair of interacting identical spherical galaxies on their separation is studied using polytropic models for the galaxies. It is found that if the escape (parabolic) velocity of the pair is expressed in units of $V_{\rm rms}$ of the stars and the separation of the galaxies in units of the mean radius of the galaxy, the relationship between these two quantities is independent of the mass distribution of the galaxies. This relationship is given.

Key words : interacting galaxies

1. Introduction

A knowledge of escape velocities of interacting galaxies is useful in studies of dynamics of interacting galaxies. Using the analysis of spherically symmetric matter by Limber (1961), Alladin (1965) developed a theory for determining the interaction potential energy of two galaxies treated as superposition of polytropes and tabulated the functions needed to calculate the potential energy. In the present paper this theory is used to obtain a relationship between the parabolic velocity of a pair of spherical galaxies and their separation which holds for all density distributions.

2. Escape velocity of galaxies

We represent the mass distribution in the two galaxies of masses M_1 and M_2 by polytropic distributions of common radius R and of indices n_1 and n_2 respectively. It follows from the theory of polytropes that the interaction potential energy W of a pair of galaxies is (Alladin 1965)

$$W(s) = -\frac{GM_1M_2}{R}\,\frac{\Psi(n_1, n_2, s)}{s} \qquad \ldots(1)$$

where $s = r/R$, r being the distance between the centres of the two galaxies. The escape (parabolic) velocity $V_e^{(1)}$, of the two galaxies can be obtained by setting the energy due to orbital motion of the galaxies equal to zero. This gives

$$\frac{1}{2}\frac{M_1 M_2}{M_1 + M_2}[V_{\mathrm{e}}^{(1)}]^2 = -W. \qquad ...(2)$$

Using equations (1) and (2) we obtain

$$V_{\mathrm{e}}^{(1)} = \left[\frac{2G(M_1 + M_2)}{R}\frac{\Psi(n_1, n_2, s)}{s}\right]^{1/2}. \qquad ...(3)$$

The function $\Psi(n_1, n_2, s)$ has been tabulated by Alladin (1965). Escape velocities for a pair of galaxies of the same size having any mass distribution can be obtained from this relation. To obtain escape velocities of a pair of spherical galaxies of unequal size, the function Ψ/s tabulated by Potdar & Ballabh (1974) may be used.

We measure escape velocity in units of the root mean square internal speed V_{rms} for the galaxy of mass M, obtained from the virial theorem and the self potential energy, Ω, of the galaxy having density distribution of the polytrope of index n. Thus

$$MV_{\mathrm{rms}}^2 = -\Omega = \frac{3}{5-n}\frac{GM^2}{R} \qquad ...(4)$$

We express the separation between the two galaxies in terms of the mean radius $\bar{R}$, obtained from the relation (Chandrasekhar 1942):

$$\Omega = -GM^2/2\bar{R}. \qquad ...(5)$$

It can be readily seen that $V_{\mathrm{e}}^{(1)}/V_{\mathrm{rms}} = 2.82$ at $r = 0$ for all polytropic models of the same mass and radius. We have

$$W = -2\frac{3}{5-n}\frac{GM_1M_2}{R} \quad (r = 0, n_1 = n_2 = n) \qquad ...(6)$$

and hence for $M_1 = M_2$ we get from equations (5) and (6)

$$\frac{W}{\Omega} = 2 \ (r = 0). \qquad ...(7)$$

Using equations (2) and (4), we get for a pair of identical galaxies

$$V_{\mathrm{e}}^{(1)}/V_{\mathrm{rms}} = 2\sqrt{2} \quad (r = 0). \qquad ...(8)$$

The present work shows that the relationship between $V_{\mathrm{e}}^{(1)}/V_{\mathrm{rms}}$ and $r/\bar{R}$ is independent of n for all values of $r/\bar{R}$ for all pairs of identical galaxies.

In Figure 1 we plot the value of $V_{\mathrm{e}}^{(1)}/V_{\mathrm{rms}}$ as a function of $r/\bar{R}$ for a pair of galaxies having identical mass distribution ($n_1 = n_2$) and the same radius. We find that the plots for $n = 0, 1, 2, 3$ and 4 give the same curve. This makes it very convenient to obtain the escape velocity of any pair of identical spherical galaxies. The following polynomial formula

$$\frac{V_e^{(1)}}{V_{\rm rms}} = 2.82 - 0.307\,(r/\bar{R}) - 0.0379\,(r/\bar{R})^2,\ 0 \leqslant r/\bar{R} \leqslant 3 \qquad \ldots(9)$$

describes the relationship quiet well in the range of interpenetration. The probable error is $\pm\,0.035$.

For non-interpenetrating galaxies, $\Psi = 1$ in equation (3). Using equations (3), (4) and (5) we obtain

$$\frac{V_e^{(1)}}{V_{\rm rms}} = 2\sqrt{2}\,(r/\bar{R})^{-1/2}, \quad r/\bar{R} \geqslant 3. \qquad \ldots(10)$$

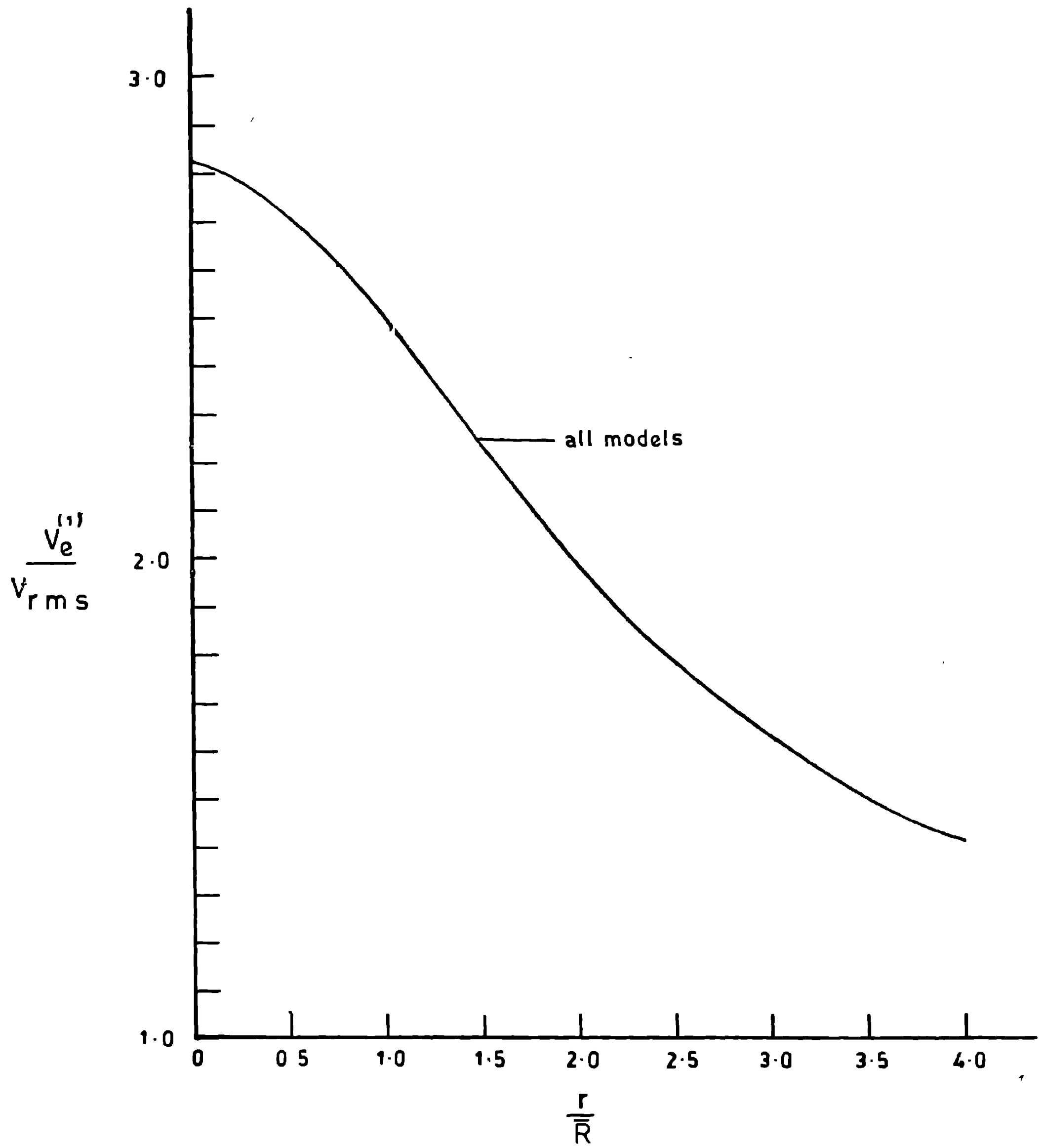

Figure 1. Escape (parabolic) velocity, $V_e^{(1)}$, of a pair of galaxies.

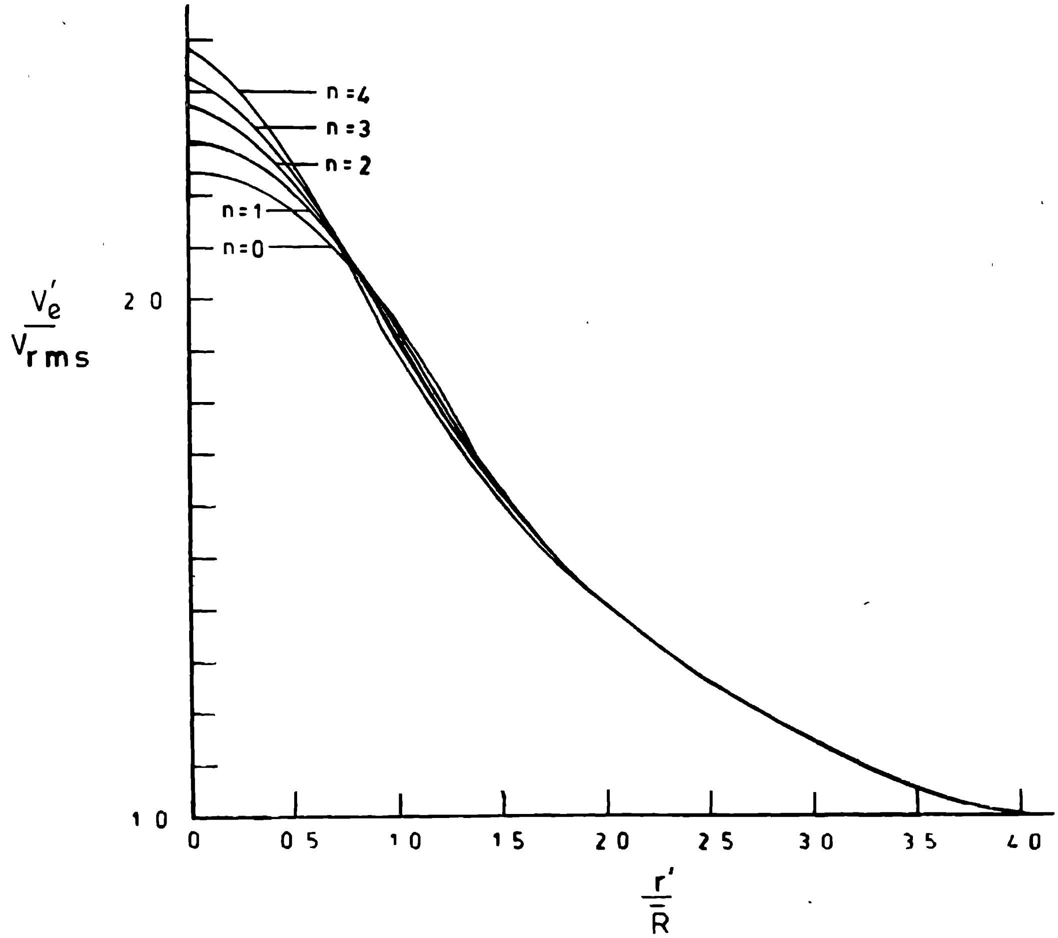

Figure 2. Escape velocity, V_e', of a star in a galaxy.

3. Discussion

It should be noted that a plot of $V_e'/V_{\rm rms}$ against $r'/\bar{R}$, where V_e' is the escape velocity of a star in a galaxy situated at a distance r' from the centre of the galaxy, is not the same for all polytropic models if $r'/\bar{R} < 1$. This is illustrated in Figure 2 for $n = 0, 1, 2, 3$ and 4. Although $n = 0$ and 1 are not appropriate for models of galaxies, we have included these for the sake of comparison. V_e' is obtained as discussed in Sastry & Alladin (1970).

The escape (parabolic) velocity of galaxies derived from equation (3) is actually a lower limit to the actual value, since the decrease in the energy associated with the orbital motion of the galaxies due to an increase in the internal energy of the two galaxies during collision is not taken into account. Making allowance for this decrease in energy during the encounter, Sastry (1972) made a better estimate, $V_e^{(2)}$, for the velocity of escape. It follows from his results that the ratio of $V_e^{(2)}/V_e^{(1)}$ for a pair of galaxies treated as polytropes of index $n = 4$ in a head-on collision is equal to 1.16. This is in agreement with the value obtained by

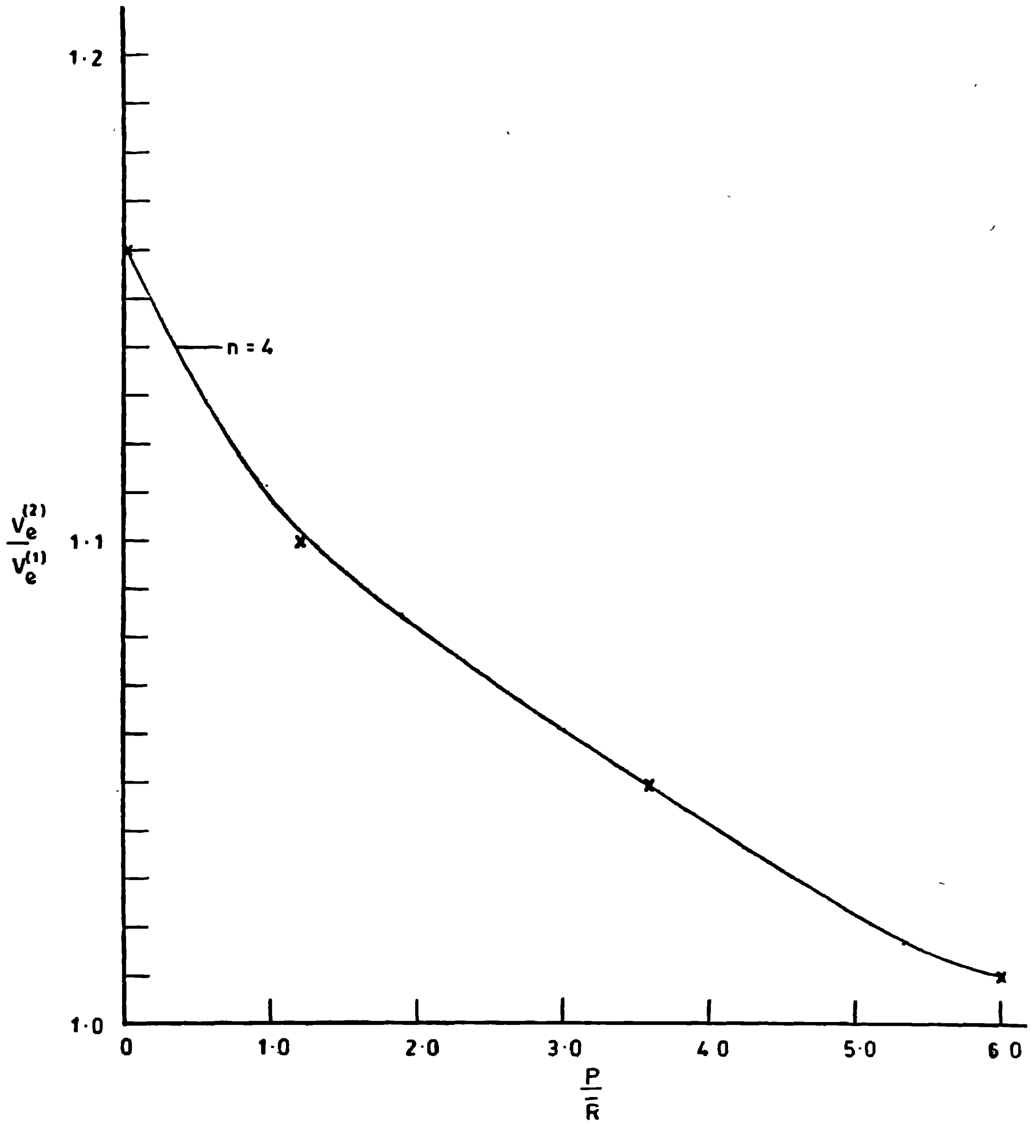

Figure 3. $V_e^{(2)}/V_e^{(1)}$ for a pair of galaxies.

van Albada & van Gorkom (1977) with polytropes $n = 3$ model for galaxies using N-body simulations and by Toomre (1977) with Plummer model for galaxies using the impulsive approximation. Roos & Norman's (1979) results based on N-body simulations are also consistent with this value.

In Figure 3, we have plotted $V_e^{(2)}/V_e^{(1)}$ derived from Sastry (1972) for identical galaxies represented by polytrope $n = 4$ model. This curve depends only slightly on the chosen model of the galaxies. Roos & Norman (1979) give the critical velocity ($V_e^{(2)}$ in our notation) as a function of separation of galaxies for galaxies with massive halos. The drop in the critical velocity with separation is less steep in their model.

For $p \gg \bar{R}$ where p is the distance of closest approach of the two galaxies, an analytic expression for $V_e^{(2)}/V_e^{(1)}$ can be obtained from the impulsive approximation. The loss in orbital energy ΔE in a complete encounter of two identical galaxies in this case is (Tremaine 1980).

$$\Delta E = \frac{8G^2M^3 \langle r^2 \rangle}{3V^2p^4}, \qquad \text{...(11)}$$

where M is the mass of each galaxy and V their relative speed. Setting

$$\tfrac{1}{2}\mu \, [V_e^{(2)}]^2 - \tfrac{1}{2}\mu \, [V_e^{(1)}]^2 = \tfrac{1}{2}\Delta E, \qquad \text{...(12)}$$

where μ is the reduced mass, we obtain

$$\frac{V_e^{(2)}}{V_e^{(1)}} \approx 1 + \frac{1}{6} \frac{\langle r^2 \rangle}{p^2}. \qquad \text{...(13)}$$

$[\langle r^2 \rangle]^{1/2} = 0.772\,R$ for a homogeneous sphere ($n = 0$) and $0.188\,R$ for a polytropic sphere of index $n = 4$ where R is the radius of the sphere. Equation (13) may be used to obtain the critical velocity at large distances.

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