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LIMITS TO THE MASS AND THE RADIUS OF THE COMPACT STAR IN SAX J1808.4−3658 AND THEIR IMPLICATIONS

Sudip Bhattacharyya[1,2]
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ABSTRACT

We show that a survey of equations of state and observations of X-ray pulsations from SAX J1808.4−3658 give 2.27 $M_\odot$ as the upper limit of the compact star mass. The corresponding upper limit of the radius comes out to be 9.73 km. We also do a probabilistic study to estimate the lower limit of the mass of the compact star. Such a limit puts useful constraints on equations of state. We also discuss the implications of the upper mass limit for the evolutionary history of the source, as well as the detection of it in radio frequencies. We envisage that the possible observation of radio eclipse may be able to rule out several soft equation-of-state models, by setting a moderately high value for the lower limit of the inclination angle.

Subject headings: accretion, accretion disks — binaries: close — equation of state — pulsars: individual (SAX J1808.4−3658) — X-rays: stars

1. INTRODUCTION

The discovery of millisecond X-ray pulsations (period $T = 2.49$ ms; Wijnands & van der Klis 1998) in the transient X-ray burster SAX J1808.4−3658 confirmed the speculation that low-mass X-ray binaries are progenitors of millisecond pulsars (Bhattacharya & van den Heuvel 1991). The orbital period ($P_{\rm orb} = 2.01$ hr) and the pulsar mass function ($f_1 = 3.7789 \times 10^{-5}$) of this source were observationally determined by Chakrabarty & Morgan (1998). These give valuable information about the masses (of both the primary and the secondary) and the inclination angle. For example, the value of $P_{\rm orb}$ uniquely determines the mass of a Roche lobe–filling low-mass star with a known mass-radius relation.

It has been recently proposed that the compact star in SAX J1808.4−3658 is a strange star (SS) and not a neutron star (NS) (Li et al. 1999). Such a speculation, if confirmed, will prove that the so-called *strange matter hypothesis* (Witten 1984) is correct. According to this hypothesis, strange quark matter (made entirely of deconfined *u*, *d*, and *s* quarks) could be the true ground state of strongly interacting matter rather than ^{56}Fe. This is an important problem of the fundamental physics. To resolve it, we need to constrain the equations of state (EOSs) for this compact star very effectively.

In this Letter, we estimate the upper limits of the mass and the radius of the compact star in SAX J1808.4−3658. We also discuss the possible ways to estimate the lower mass limit.

2. UPPER LIMITS TO MASS AND RADIUS

We estimate the upper limits of the mass and the radius of the compact star in SAX J1808.4−3658 by using the basic requirements for X-ray pulsations. Here, up to equation (5), we follow the same method as described in Li et al. (1999). To explain it, we first define the corotation radius ($R_{\rm co}$) and the magnetospheric radius ($R_{\rm mag}$). They are given by (see Burderi & King 1998)

$$R_{\rm co} = 1.5 \times 10^6 m_1^{1/3} T^{2/3}, \qquad (1)$$

$$R_{\rm mag} = 1.9 \times 10^6 \phi \mu_{26}^{4/7} m_1^{-1/7} \dot{M}_{17}^{-2/7}, \qquad (2)$$

where m_1 is the compact star mass in units of solar mass, T is the compact star spin period in units of milliseconds, ϕ is the ratio between the magnetospheric radius and the Alfvén radius, μ_{26} is the compact star magnetic moment in units of 10^{26} G cm^3, and $\dot{M}_{17}$ is the accretion rate in units of 10^{17} g s^{-1}; $R_{\rm co}$ and $R_{\rm mag}$ are given in centimeters. In this Letter, we assume that ϕ is almost independent of the accretion rate (Burderi & King 1998).

The requirements for X-ray pulsations (if there is no "intrinsic" pulse mechanism) and the presence of accretion flow (that is not centrifugally inhibited) give (see Li et al. 1999)

$$R_1 < R_{\rm mag}(\dot{M}_{\rm max}) < R_{\rm mag}(\dot{M}_{\rm min}) < R_{\rm co}, \qquad (3)$$

where $\dot{M}_{\rm min}$ and $\dot{M}_{\rm max}$ give the range of the accretion rate in which X-ray pulsations in SAX J1808.4−3658 were observed. From equations (1)–(3), we get (Li et al. 1999)

$$R_1 < 27.6 \left(\frac{F_{\rm max}}{F_{\rm min}}\right)^{-2/7} \left(\frac{T}{2.49\ {\rm ms}}\right)^{2/3} m_1^{1/3}\ {\rm km}, \qquad (4)$$

where $F_{\rm max}$ and $F_{\rm min}$ are the maximum and minimum values of measured X-ray fluxes, respectively. It is to be noted that here the pulsar magnetic field is assumed to be dipolar. In writing equation (4), we also assume that $\dot{M}$ is proportional to the observed flux F for all accretion rates. This is justified by the fact that the X-ray spectrum of SAX J1808.4−3658 was remarkably stable (Gilfanov et al. 1998), when the X-ray luminosity varied by a factor of ∼100 during the 1998 April/May outburst. During this period, in the 2–30 keV band, the maximum observed flux was around 3×10^{-9} ergs cm^{-2} s^{-1}, while the flux value dropped to around 2×10^{-11} ergs cm^{-2} s^{-1} when the pulse signal became barely detectable (Cui, Morgan, & Titarchuk 1998). Adopting

[1] Joint Astronomy Program, Indian Institute of Science, Bangalore 560 012, India; sudip@physics.iisc.ernet.in.
[2] Indian Institute of Astrophysics, Bangalore 560 034, India; sudip@iiap.ernet.in.

TABLE 1
13 EOSs of Widely Varying Stiffness Parameters, Their References, and Values of Relevant Properties (See the Text)

EOS Label	Compact Star	Reference	$m_{1,\max}$	R_1 (km)	x_1
SS2	SS	1	1.32	6.53	3.34
SS1	SS	1	1.44	7.07	3.32
B_{90}	SS	2	1.60	8.74	3.69
B_{60}	SS	3	1.96	10.71	3.70
Y	NS	4	1.41	7.10	3.39
B	NS	5	1.79	9.64	3.64
W	NS	6	2.28	11.22	3.32
SBD	NS	7	2.59	14.08	3.68
A	NS	8	1.66	8.37	3.42
AU	NS	9	2.13	9.41	2.98
FPS	NS	10	1.80	9.28	3.48
L	NS	11	2.70	13.70	3.43
M	NS	12	1.81	11.60	4.34

References.—(1) Dey et al. 1998. (2) Farhi & Jaffe 1984: $B = 90$ MeV fm^{-3}, $m_s = 0$. (3) Farhi & Jaffe 1984: $B = 60$ MeV fm^{-3}, $m_s = 0$. (4) Pandharipande 1971b: hyperonic matter. (5) Baldo, Bombaci, & Burgio 1997: nuclear matter. (6) Walecka 1974: neutron matter. (7) Sahu, Basu, & Datta 1993: nuclear matter. (8) Pandharipande 1971a: Reid soft core. (9) Wiringa, Fiks, & Fabrocini 1988: AV14 + UVII. (10) Lorenz, Ravenhall, & Pethick 1993: UV14 + TNI. (11) Pandharipande & Smith 1975b: mean field. (12) Pandharipande & Smith 1975a: tensor interaction.

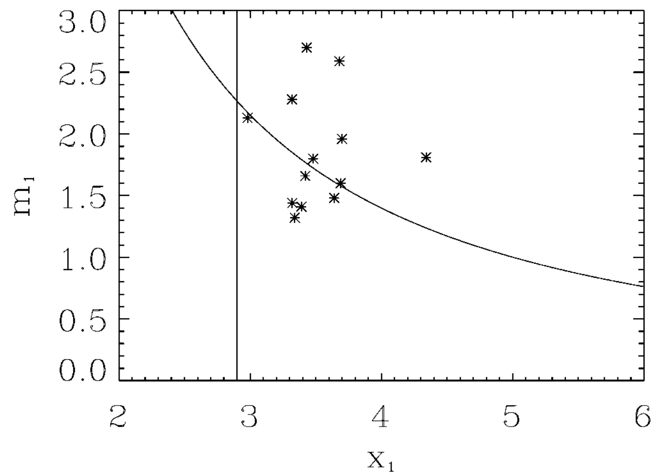

Fig. 1.—Plot of m_1 vs. x_1 (see the text) for a compact star. The solid curve indicates the upper bound of the mass according to eq. (6). The asterisks are for different EOS models listed in Table 1. The vertical line corresponds to $x_1 = 2.9$.

the maximum-to-minimum flux ratio as 100 (after Li et al. 1999), we get from equation (4)

$$R_1 < 7.40 m_1^{1/3}\ \text{km}. \tag{5}$$

Equation (5) gives the maximum value of R_1, if the maximum value of m_1 is known. To calculate $m_{1,\max}$ we first rewrite equation (5) in the following form:

$$m_1 < 11.19 x_1^{-3/2}, \tag{6}$$

where x_1 is the dimensionless radius-to-mass ratio of the compact star. We can compute $m_{1,\max}$ from equation (6), if the minimum value of x_1 is known. To choose the value of $x_{1,\min}$, we survey about 20 EOSs (that include both SSs and NSs) and examine the value of x_1 corresponding to the maximum possible mass for a given EOS. For both SSs and NSs, we choose EOSs of widely varying stiffness parameters, which guarantees our results to be of sufficient generality. This is reflected by the wide range of maximum possible mass values given in Table 1, where we have listed 13 representative EOSs. From Table 1 and Figure 1, we notice that the x_1-values for all the EOSs are confined to the range 2.98–4.34, with 11 (out of 13) points clustering in 3.3–3.7. To illustrate this, we draw a vertical line in Figure 1, corresponding to $x_1 = 2.9$. As none of the EOS points fall to the left of this line, we take 2.9 as the lower limit of x_1. Such a conclusion is very general, as it is valid for the whole range of existing EOSs. This gives 2.27 (the crossing point of the vertical line and the curve in Fig. 1) as the upper limit of m_1 from equation (6). The corresponding upper limit of R_1 comes out to be 9.73 km from equation (5).

It is to be noted that for some SS EOSs, the x_1-value may be less than 2.9 for lower values of masses (i.e., less than the maximum possible mass). But, as we use the lower limit of x_1 to estimate the maximum possible value of m_1, it is justified to take 2.9 as the minimum possible value of x_1. An EOS model (that may be put forward in future) with x_1 (corresponding to the maximum possible mass) less than 2.9 will give a higher value of $m_{1,\max}$ than 2.27. However, such an unusual EOS is highly improbable. We also point out that if we take into account the rotation of the compact star, the lower limit of x_1 will increase, resulting in a decrease of $m_{1,\max}$. Therefore, we can say that 2.27 may be the firm upper limit of m_1.

For the sake of completeness and to give more credibility to our work, we calculate $m_{1,\max}$ with less constraining values of x_1. For this purpose, we take $x_{1,\min} = 2.25$, which is the absolute lower limit (for a compact star) of x_1 (Weinberg 1972). This limit, which is independent of EOSs and depends only on the structure of the relativistic equations for hydrostatic equilibrium, gives $m_{1,\max} = 3.32$ and $R_{1,\max} = 11.04$ km. Therefore, 3.32 is the absolute upper limit of m_1. Another value of $x_{1,\min}(=2.56)$ was derived by Bondi (1964), under the reasonable assumptions concerning the EOS, i.e., $\epsilon > 0$, $p > 0$, and $dp/d\epsilon < 1$ (where p is pressure and ϵ is energy density). This value of $x_{1,\min}$ implies $m_{1,\max} = 2.73$ and $R_{1,\max} = 10.34$ km. Therefore, we see that the value of $R_{1,\max}$ is not very sensitive to the chosen value of $x_{1,\min}$.

3. LOWER MASS LIMIT

Here we estimate the probability ($P_{\min}$) of a possible compact star mass (m_1) to be the lower limit of mass. We do it using the "random distribution of orbital inclinations" procedure for measuring NS mass, mentioned in Thorsett & Chakrabarty (1999). Because of the absence of sufficient observational data, here we cannot follow any well-established statistical method. For example, the measured value of a single post-Keplerian parameter (Taylor 1992; with additional assumptions, such as a uniform prior likelihood for orbital orientations with respect to the observer) can be used to make strong statements about the posterior distribution of the masses (Thorsett & Chakrabarty 1999). But none of these parameters could be measured for SAX J1808.4−3658. Therefore, our results basically depend on the a priori probability of observing the source with a given inclination angle (i).

To explain the method, we first rewrite the well-known expression for the pulsar mass function (f_1) in the following way:

$$\sin i = f_1^{1/3} \frac{(m_1 + m_2)^{2/3}}{m_2}, \tag{7}$$

where m_2 is the mass of the companion star in units of solar mass. For a main-sequence companion that fills its Roche lobe, $m_2 = 0.22$ (corresponding to $P_{\rm orb} = 2.01$ hr). As a result, the lower limit of i (i.e., $i_{\rm min}$) comes out to be 3° from equation (7) (using $m_{1,\,\rm min} = 0$, the absolute lower limit). However, Chakrabarty & Morgan (1998) have argued that $m_2 \lesssim 0.1$ (because the companion is bloated by irradiation). Therefore, we take 0.1 as the upper limit of m_2, which corresponds to $i_{\rm min} = 4°$. The absence of a deep eclipse indicates that for a Roche lobe–filling companion, $i_{\rm max} = 82°$ (Chakrabarty & Morgan 1998). We set $m_{2,\,\rm min} = 0.05$, which is a possible companion mass according to Chakrabarty & Morgan (1998). We also take two other values of $m_{2,\,\rm min}$ for the purpose of illustration.

With all these limiting values, we calculate $P_{\rm min}$ in the following way. Given a value of m_1, we compute the allowed range of i (i.e., $i_{a,\,\rm min}$ and $i_{a,\,\rm max}$) from equation (7), for the chosen range of m_2 (i.e., $m_{2,\,\rm min} \le m_2 \le m_{2,\,\rm max}$). However, for $i_{a,\,\rm max} > i_{\rm max}$, we take $i_{a,\,\rm max} = i_{\rm max}$. Similarly, we do not consider any value of i less than $i_{\rm min}$. Now we argue (after Chakrabarty & Morgan 1998) that in statistical calculations it is useful to assume that binary orbits are randomly oriented with respect to the line of sight (see also Thorsett & Chakrabarty 1999). The differential distribution of inclinations is then proportional to $\sin i$. This gives the a priori probability of observing a system with i in the range $i_{a,\,\rm min} \le i \le i_{a,\,\rm max}$ as $P = (\cos i_{a,\,\rm min} - \cos i_{a,\,\rm max})$. Therefore, P should be the probability of the chosen m_1 for being the actual compact star mass.

We calculate P for many m_1-values (at regular intervals) in the range $m_{1,\,\rm min} \le m_1 \le m_{1,\,\rm max}$. Then $P_{\rm min}$ is calculated by the formula

$$P_{\rm min} = \frac{\sum_{i=j}^{n} P_i}{\sum_{i=1}^{n} P_i}, \tag{8}$$

where P is calculated at n number of m_1 points and j is the index number of the m_1-value at which $P_{\rm min}$ is required.

In Figure 2, we plot $P_{\rm min}$ against m_1 for three values of $m_{2,\,\rm min}$ (0.04, 0.05, and 0.08). For $m_{2,\,\rm min} = 0.05$, we see that the minimum value of m_1 is 0.70 with 95% probability. Therefore, although we start with zero as the lower limit of m_1, we get a large value for $m_{1,\,\rm min}$ with a very high probability. This shows that the probabilistic method is very effective in estimating the value of $m_{1,\,\rm min}$. If we take $m_{1,\,\rm max} = 3.32$ (i.e., the absolute limit), the minimum value of m_1 comes out to be 0.90 with 95% probability, which shows that this method is sensitive to the assumed value of $m_{1,\,\rm max}$ (a less constrained value of the upper limit of m_1 gives a higher value of $m_{1,\,\rm min}$ with the same probability). A considerable increase in either $m_{2,\,\rm max}$ or $i_{\rm max}$ does not change our result much. For example, if $m_{2,\,\rm max} = 0.22$, the 95% probabilistic minimum value of m_1 is 0.68. However, our result changes with the change of $m_{2,\,\rm min}$ for $m_{2,\,\rm min} < 0.04$.

We also point out that for a given value of $m_{2,\,\rm min}$, if $i_{\rm min}$ is greater than a certain value, it can be seen from equation (7) that every $i_{\rm min}$ will correspond to a minimum possible value of m_1. Therefore, if we can observationally constrain i from the lower side, the value of $m_{1,\,\rm min}$ can be predicted more accurately (as it will not depend on the probabilistic study). For example, the detailed modeling of the optical companion's multiband photometry during outburst with a simple X-ray–heated disk model suggests that $\cos i < 0.45$ (Y.-M. Wang et al. 2001, in preparation; Bildsten & Chakrabarty 2001) for SAX J1808.4−3658. This implies $i > 63°$ and hence $m_{1,\,\rm min} = 1.48$ (using $m_{2,\,\rm min} = 0.05$).

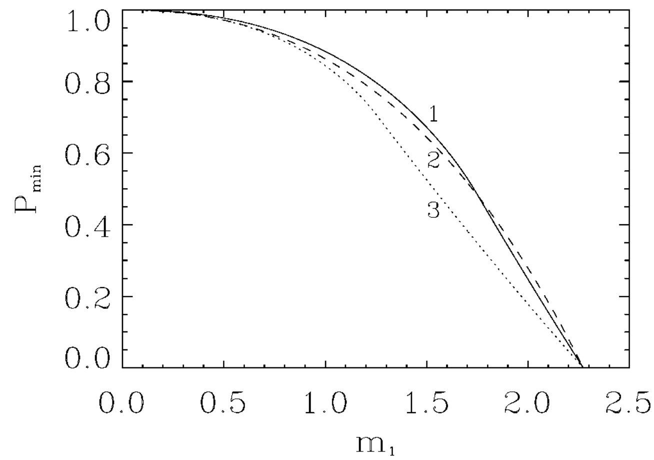

FIG. 2.—Plot of $P_{\rm min}$ vs. m_1 with the following parameter values: $m_{1,\,\rm min} = 0$, $m_{1,\,\rm max} = 2.27$, $m_{2,\,\rm max} = 0.1$, $i_{\rm min} = 4°$, and $i_{\rm max} = 82°$. Curve 1 is for $m_{2,\,\rm min} = 0.05$, curve 2 for $m_{2,\,\rm min} = 0.08$, and curve 3 for $m_{2,\,\rm min} = 0.04$.

4. DISCUSSION

In this Letter, we have estimated the upper limits of the mass and the radius of the compact star in SAX J1808.4−3658. Li et al. (1999) have concluded that a narrow region in m_1-R_1 space will be allowed for this star. The upper boundary of the mass will constrain this region effectively. It can also give the upper limit of i (from eq. [7]), if $m_{2,\,\rm min}$ is known by an independent measurement. Alternatively, $m_{1,\,\rm max}$ gives the upper limit of m_2, for a known value of $i_{\rm min}$. For example, $i_{\rm min} = 63°$ gives $m_{2,\,\rm max} = 0.066(0.085)$ for $m_{1,\,\rm max} = 2.27(3.32)$.

As we have mentioned in § 2, in this work, the pulsar magnetic field is assumed to be primarily dipolar. If the field has more complicated structure, the $R_{\rm mag}$-$\dot{M}$ relation will be changed, resulting in the modification of equation (4). This will lead to the change in equation (6), and hence our calculated value of $m_{1,\,\rm max}$ (and $R_{1,\,\rm max}$) will be modified. However, Li et al. (1999) have argued that the accretion flow around the compact star is dominated by a central dipole field, which gives credibility to our results.

Corresponding to every EOS, there exists a maximum possible mass. Therefore, a lower limit of m_1 is very important in constraining EOSs and hence in understanding the properties of matter at a very high density compact star core. The possibility of this candidate to be an SS can also be checked more effectively. Using our Figure 2, we can predict the probability with which a certain value of m_1 will be the lower limit. For example, $m_1 = 1.41$ (the maximum possible mass for our model Y) will be the lowest possible mass with 72% probability (from curve 1 of Fig. 2). However, it is to be kept in mind that such a probabilistic study may be valid, if binary inclination angles are distributed randomly.

If the orbital evolution of SAX J1808.4−3658 is driven by only gravitational wave radiation, then the rate of change of the orbital period will be given by (Ergma & Antipova 1999)

$$\dot{P}_{\rm orb} = -1.72 \times 10^{-7} (2\pi/P_{\rm orb})^{5/3} m_2 m_1 (m_1 + m_2)^{-1/3}. \quad (9)$$

This implies 2.30×10^{-13} as the maximum possible (absolute) value of $\dot{P}_{\rm orb}$ (for $m_{1,\max} = 2.27$, $m_{2,\max} = 0.1$, and $P_{\rm orb} = 7249$ s). Chakrabarty & Morgan (1998) have suggested that $\dot{P}_{\rm orb}$ can be measured, if the source remains in the X-ray bright state for long enough (or if the pulsations remain detectable in quiescence). If in the future such a measurement yields the value of the orbital period decay rate greater than 2.30×10^{-13} (2.99×10^{-13} for $m_{1,\max} = 3.32$), then we can conclude with a certain confidence that the orbital evolution of SAX J1808.4−3658 is significantly driven by magnetic braking. This will give support to the evolutionary model of Ergma & Antipova (1999) and in general will be very important for learning about the prehistory of the system. A better understanding of the criterion for magnetic braking will also be possible.

It has been proposed that SAX J1808.4−3658 may emerge as a radio pulsar during the X-ray quiescence (Chakrabarty & Morgan 1998). Ergma & Antipova (1999) have calculated that for $\lambda < 3$ cm, it may be possible to observe radio emission from this source. However, our limits of mass values give a slightly higher (3.8 cm for $m_{1,\max} = 2.27$ and 4.5 cm for $m_{1,\max} = 3.32$) upper limit for λ.

As we have already mentioned in § 3, a moderately high value of $i_{\min}$ will give a lower limit of m_1 (without any probabilistic study). This will be very important for constraining EOSs more decisively. For example, if $i_{\min} = 63°$ (corresponds to $m_{1,\min} = 1.48$, given in § 3), our EOS models SS1, SS2, and Y will be unfavored (see the "$m_{1,\max}$" column of Table 1). According to Chakrabarty & Morgan (1998), a deeper eclipse might be observed for the less penetrating radio emission, providing a strong constraint on the value of i. Therefore, we expect that the value of $i_{\min}$ (determined by this method) may be able to rule out several soft EOS models in the future.

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