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On the Resolving Power of an Echelle Grating in the Presence of a Corrugated Wavefront of Incidence

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The resolving power of a grating is determined by the ability of the system to separate the intensity maxima produced by two infinitesimally close wavelengths. The design parameters of a grating address the two issues - (a) improvement of the resolving power and (b) enhancement of dispersive property[1]. An echelle grating can ideally satisfy these criteria giving a resolving power $\sim 10^6 - 10^7$. This high resolution property presupposes that the wave front of incidence is perfectly coherent. The question asked here is – what happens to the resolving power of the grating if the phase coherence in the incidence beam be lost beyond a distance $\rho_o = \lambda/2\pi\theta_s$, where, θ_s is the 'seeing' angle. When incident on the grating however, the coherence is compressed to a distance r_o by collimation, where $r_o = (d/D)\rho_o$, where D is the diameter of the telescope and 'd' being the length of the grating. We consider the scattering process from the echelle grating to be equivalent to that from a "periodic rough surface"[2] and in terms of the standard definitions therein, we have the scattering wave vectors,

$$\nu_x = k(\sin\theta_1 - \sin\theta_2\cos\theta_3);\ \nu_z = -k(\cos\theta_1 + \cos\theta_2) \tag{1}$$

with $k = 2\pi/\lambda$; and λ being the wavelength of light with the groove being in x direction and scattering in y direction is absent. By addition of the complex amplitude of all the rays diffracted by the grating with N groove of width '$2a$' each and blazing angle α, we find the intensity to follow,

$$\begin{aligned}\langle EE^*\rangle \;=\; & \sum_{n=-N}^{N}\sum_{m=-N}^{N}\int\int\langle e^{i[\phi_n(x_1,y_1)-\phi_m(x_2,y_2)]}\rangle \times \\ & e^{2ia\nu_x(n-m)}e^{i(\nu_x+\nu_z\tan\alpha)(x_1-x_2)}dx_1dx_2\end{aligned} \tag{2}$$

where, $\phi_n(x,y)$ is the random phase at the point of incidence $(2na + x)$ on the nth groove of the grating. Considering $\phi_n(x,y)$ to be gaussian stationary random phases and assuming that

$$\langle[\phi_n(x_1,y_1) - \phi_m(x_2,y_2)]^2\rangle = \left\{[2a(n-m) + (x_1 - x_2)]^2 + (y_1 - y_2)^2\right\}/r_o^2 \tag{3}$$

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i.e. the Fried exponent 5/3 is replaced by 2, we are able to approximate $\langle EE^* \rangle$ by a simple sum involving a trigonometric and hyperbolic functions, which are amendable to simple computations. Analytical calculations introduce an effective-coherence length r_*, given by

$$r_*^4 = 16\left[N^2a^2r_o^2/6\right] / \left[(r_o^2/4) + 2N^2a^2/9)\right] \tag{4}$$

In Fig. 1(a) we give the intensity pattern for λ = 5890Å and show that the pattern changes as the seeing angle θ_s changes. The maxima are seen to broaden as θ_s is increased.

In all the results displayed here, we have chosen $2a$ = 1900μm; $\alpha = 70^o$; and N = 21400 which are certain typical cases. In Fig. 1(b) we show how the resolution of the two lines λ_1 = 5890Å and λ_2 = 5894Å is affected as the seeing angle θ_s is changed. This is related to the fact that the resolving power, $R = \lambda/\delta\lambda \approx (\kappa)r_*\lambda$, where $\kappa \cong 66$ according to Rayleigh criterion and $\kappa \cong 8.88$ according to the Sparrow criterion of resolution[3]. Since the resolving power goes as $R = (\kappa)r_*/\lambda$, we plot R versus θ_s in Fig. 1(c). From this curve the resolving power for any arbitrary θ_s and for any κ (for the Rayleigh or the Sparrow criterion) can be easily calculated. The R versus θ_s shows that the resolvability is decided by the coherence properties of the incident wave front i.e. $r_* = 4Na$ if $r_o >> 4Na$ and $r_* \sim r_o$ if $r_o << 4Na$. In other words, resolution is dominated by the resolving power of the worse component of the system, demonstrating the need for proper wave front correction if the full resolvability of the grating be exploited for spectroscopy of astronomical sources.

In a simple experiment, we have verified the above results by distorting the incident wavefront and checking the dispersion of an accurately known profile, –being the 6328Å of the He-Ne laser –as seen by an imperfect spectroscope. We produced phase diffusers by etching glass plates with hydrofluoric acid for different time intervals t. It was noted that when the He-Ne laser beam was passed through the diffuser the broadening of the beam was proportional to the etching time t. The He-Ne light on passing through the diffuser, when analyzed by a S200 fibre optic spectrometer (resolution 0.3nm) was shown to give a profile, which broadens linearly with the time of etching. This is shown in Fig. 2(a) and Fig. 2(b), respectively, as proofs of theory given above.

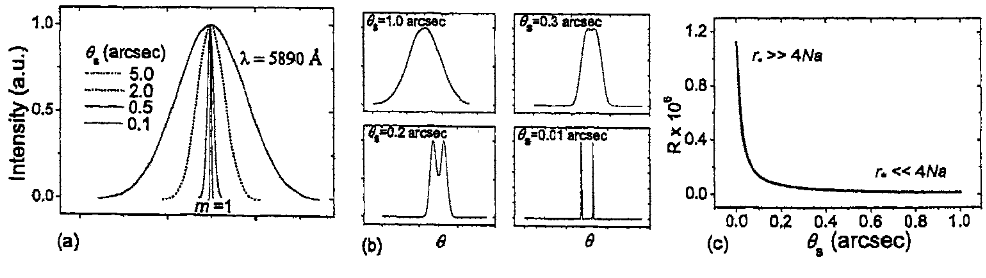

Figure 1. (a) Broadening of the intensity profile with increasing θ_s (b) Effect of θ_s on the resolution of closely spaced sodium lines (c) Plot of resolving power R versus seeing angle θ_s

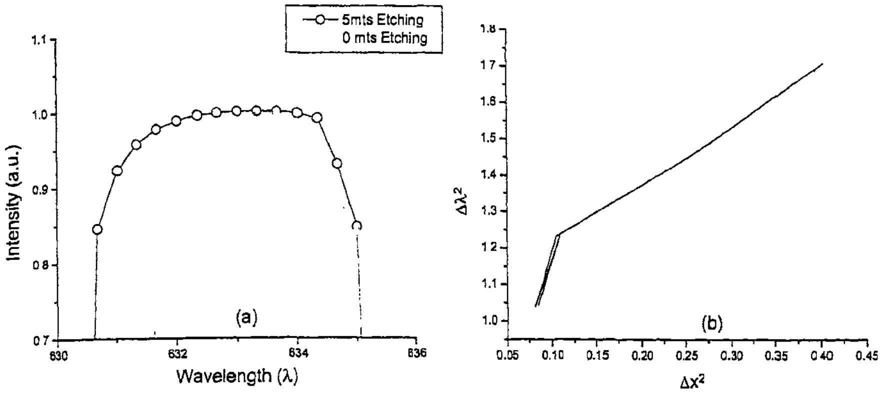

Figure 2. (a) He-Ne laser line profile for different diffusers in the path (b) the width of the spectral line of the He-Ne laser with etching time.

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