Linear sizes and arm asymmetry of intrinsically symmetric doubles

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Abstract. In an intrinsically symmetric model, the distribution \( p_\phi(x) \) of the fractional arm differences \( x \) of a sample of straight double radio sources reflects the distribution among the doubles of their orientations \( \phi \) away from the line of sight. In a constant speed expansion model, the two distributions can be simply related, and the resultant distribution \( p_l(l) \) of the projected linear sizes \( l \) can be derived, to be compared with the observed distribution for the same sample. Such a calculation is illustrated using a sample of extended doubles. The model is perhaps more applicable to compact doubles. However, these are not yet numerous enough to construct usable samples.

1. Model calculations

Orientations from Arm Asymmetry

Observations give \( x = (\theta_R - \theta_L) / (\theta_L + \theta_R) \); \( 0 \leq x \leq 1 \), for the fractional arm difference \( x \) of a straight double radio source having hotspots at the ends of the two arms of lengths \( \theta_R \) and \( \theta_L \). Identify the larger arm \( \theta_L \) with the arm \( \theta_A \) ending in the approaching hotspot and \( \theta_R \) with \( \theta_A \) corresponding to the receding hotspot, in a model where the two hotspots expand away at speed \( v \) (as a fraction of \( c \)) from the active galactic nucleus (AGN), along a line making angle \( \phi \) with the line of sight to the AGN. Denoting \( \cos \phi \) by \( y \), we have (Banhatti 1980) \( x = vy \), so that \( p_l(y) = p_s(vy) \) \( dx / dy = vp_y(vy) \) gives the distribution of orientations in terms of arm asymmetry as parametrized by \( x : x = 0 \) corresponding to a double in the sky plane and \( x = v \) to one along the line of sight. The expansion speed \( v \) is a constant parameter of the model.

2. Linear sizes

In the same model, the observed linear size \( l \) of a double is given by (Banhatti 1988) \( l = q(y)t_A \), with \( q(y) = (2v(1 - y^2)^{1/2}) / (1 + vy) \) and \( 0 \leq t_A \leq 1 \), \( t_A \) being the age (since the beginning of expansion) of the approaching hotspot. This age is taken to be distributed uniformly between 0 and some maximum, and measured as a fraction of this maximum. Hence, by the method of Banhatti (1998),

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\[ P_\ell(l) = \int_0^{\text{cutoff}} dy p_s(y) / q(y) = \int_0^{\text{cutoff}} dy p_s(vy) [(1 + vy) / 2(1 - y^2)^{1/2}] \]

(using \( p_s(y) \) derived above), where \( y_0(l) \) is given by the positive solution of the quadratic equation obtained by squaring \( l = q(y_0) \), i.e., \( l = 2v(1 - y_0^2)^{1/2}/(1 + vy_0) \) (see Banhatti 1988). This model \( p_\ell(l) \) has two parameters: \( \nu \) and \( l_{\text{max}} \) (which equals \( 2\nu \) in the units used). (Note that Banhatti (1988) evaluated the integral for \( p_\ell(l) \) using \( p_s(y) = 1; 0 \leq y \leq 1 \), corresponding to random orientations without any bias. Here, some bias is allowed by using orientations derived from arm asymmetry, i.e., \( p_s(y) = \nu p_s(vy) \).)

Details of calculating \( p_\ell(l) \) from \( p_s(x) \)

(1) For \( p_s(x) \) taken as a histogram:

\[
p_s(x) = \begin{cases} 
  p_1 & \text{for } (0 =) x_0 \text{ to } x_1, \\
  p_2 & \text{for } x_1 \text{ to } x_2, \\
  \ldots & \\
  p_n & \text{for } x_{n-1} \text{ to } x_n (= 1).
\end{cases}
\]

For each of the intervals, the indefinite integral

\[
\int dy [(1 + vy) / 2(1 - y^2)^{1/2}] = (1/2) \arcsin y - \nu(1 - y^2)^{1/2} = F(y) \text{ is needed. Using } F(y), \ p_\ell(l) = \Sigma_{i=1}^n p_i[F(x_i/v) - F(x_{i-1}/v)] + p_{n+1}[F(y_0) - F(x_n/v)], \text{ where } x_k \leq vy_0 \leq x_{k+1}, \text{ and } p_s(x > x_k) = 0, \text{ i.e., } p_s(x) \text{ has cut-off at or slightly above } x_k. \text{ Thus, } p_\ell(l) \text{ is a sum (as indicated), with terms (and their number) depending on the relative values of } x_{\text{cutoff}}, vy_0 \text{ and the break-up of } x\text{-axis into intervals in the } p_s(x) \text{ histogram.}
\]

(2) For \( p_s(x) \) as a sum of delta functions:

Inserting \( p_s(x) = (1/N) \Sigma_{i=1}^N \delta(x - x_i) \) in the integral for \( p_\ell(l) \),

\[
p_\ell(l) = (1/N) \Sigma_{i=1}^N \delta(l(x_i) - \Sigma_{j=1}^n [l(x_j)/l(x_i)]) = 2v[1 - (x/v)^2]^{1/2}/(1 + x), \text{ and } n_i = \text{number of times } x_i \text{ occurs in } p_s(x) \text{(so that } \Sigma_n n_i = N). \text{ For } x_i, i = 1, 2, \ldots, N \text{ arranged in ascending order, } p_\ell(2v) = 0, \text{ and } p_\ell(l) \text{ increases from } 0 \text{ as } l \text{ is decreased from } 2v \text{ to } 0; \text{ } p_\ell(0) \text{ being the maximum value of } p_\ell(l). \text{ At each } l(x_i), \text{ } p_\ell(l) \text{ jumps by } (n/N)(1 + x_i)/2v[1 - (x/v)^2]^{1/2} = (n/N)[1/l(x_i)]. \text{ This specifies the scheme for calculating } p_\ell(l) \text{ directly from the } x\text{-values } x_i, i = 1, 2, \ldots, N. \text{ The value of } \nu \text{ must be chosen to be } \geq \max (x_i).
\]

3. Results and discussion

Both these methods have been applied to a sample of 72 extended straight double radio galaxies listed by Best et al. (1995). See figure. The predicted (i.e., model) distribution shows a deficiency of small sizes, and a compensating excess of large sizes, indicating that some of the arm asymmetry may be intrinsic for extended doubles, presumably due to asymmetric environment over the large distances involved, the hotspots being located well outside the parent galaxy. Further, since the 3CR sample (from which our 72 sources are drawn) is flux density limited, and its radio powers and linear sizes are anticorrelated, it misses the high power large sources, explaining part of the excess of observed small sources relative to the model, which assumes the same maximum lifetime for all the sources, clearly an oversimplification.
The model is, however, more applicable to compact straight doubles, which are more likely to be intrinsically symmetric, since their hotspots from within the parent galaxy. No large enough sample is yet available, though.

As to the two computational methods used to obtain $p_l(\ell)$ from $p_x(\ell)$, the curve obtained from the histogram is essentially the smoothed 'step function' derived from the sum of delta functions. The second method is simpler and more efficient to use, and is therefore recommended.

The model has also been applied to a sample of 85 extended double radio galaxies listed by Zięba and Chyży (1991), which similar results. This sample has lower radio powers than the other one. Further work, addressing the shortcoming of the model as applied to extended doubles, and integrating these results with those on other symmetry parameters, is in progress. A more detailed version of this paper, incorporating this work, is in preparation.

![Graph](image.png)

**Figure 1.** Observed (histogram) and predicted (curve and 'step function') distributions of linear sizes for a sample of 72 extended straight double radio sources.

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**References**