

## Escape velocities of interacting galaxies in Hernquist's model

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**Abstract.** Hernquist's (1990) mass model for spherical galaxies and bulges described by deVaucouleur's  $r^{1/4}$  profile represents the observed properties of elliptical galaxies quite well. Using this model, we derive simple exact analytic formulae for the parabolic velocities of escape of two spherical galaxies and deduce analytically that for  $r = 0$ ,  $V_{\text{esc}}(0)/(V_{\text{rms}})_1 = 2\sqrt{3}$  for  $\alpha_1 \gg \alpha_2$  and  $2\sqrt{2}$  for  $\alpha_1 = \alpha_2$  and for identical galaxies for large  $r$ ,  $V_{\text{esc}}(r)/V_{\text{rms}} = 2\sqrt{2} (r/\bar{R})^{-0.5}$ . Here  $\alpha$  is the scale length,  $\bar{R}$  is the dynamical radius of a galaxy and  $r$  is the separation of the two galaxies.

### 1. Introduction

In an earlier paper (Narasimhan, Sastry and Alladin, 1997); we derived a simple and exact analytic formula for the gravitational potential energy of interacting galaxies,  $W(r)$ , in Hernquist's (1990) model. We use these results to obtain an analytic expression for the parabolic velocity of escape,  $V_{\text{esc}}$ , in this model.

### 2. Escape velocity in Hernquist's model

The basic equation which gives the escape velocity,  $V_{\text{esc}}$ , of two galaxies of masses  $M_1$  and  $M_2$  separated by a distance  $r$  is obtained by setting the energy due to their orbital motion to zero; i.e.,

$$\frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} V_{\text{esc}}^2(r) + W(r) = 0 \quad (1)$$

where  $W(r)$  is the interaction potential energy of the two galaxies. Using Equation (1) and Equation (25) of Narasimhan et al. (1997) for  $W(r)$ , we obtain

$$v_{\text{esc}}^2(r) = \frac{2G(M_1 + M_2)}{r} \left[ 1 - \frac{s_2 + \alpha_{12}}{(s_2 + \alpha_{12})^2 - 1} \alpha_{12} + 2\alpha_{12} \frac{s_2 + \alpha_{12}}{[(s_2 + \alpha_{12})^2 - 1]^2} \ln(s_2 + \alpha_{12}) \right] \quad (2)$$

where  $\alpha_1$  and  $\alpha_2$  are the scale lengths of the galaxies of masses  $M_1$  and  $M_2$  respectively,  $s_2 = r/\alpha_2$  and  $\alpha_{12} = \alpha_1/\alpha_2$ . This can be compared with the r.m.s. velocity,  $V_{\text{rms}}$ , of a galaxy of mass  $M$ , obtained by virial theorem from the self-gravitational potential energy  $\Omega$ , given by

$$MV_{\text{rms}}^2 + \Omega = 0 \quad (3)$$

from which we get

$$V_{\text{rms}}^2 = \frac{GM}{6\alpha} = \frac{GM}{2\bar{R}} \quad (4)$$

where  $\bar{R}$  is the dynamical radius.

For identical galaxies  $V_{\text{esc}}/V_{\text{rms}}$  is given by the simple expression

$$\frac{V_{\text{esc}}(s)}{V_{\text{rms}}} = \frac{2\sqrt{2}}{\sqrt{s}} \left[ 1 - \frac{3s+1}{(3s+1)^2-1} + 2 \frac{3s+1}{[(3s+1)^2-1]^2} \ln(3s+1) \right]^{1/2} \quad (5)$$

where  $s = r/\bar{R}$

$$\text{If } r \gg \bar{R}, \text{ this gives : } \frac{V_{\text{esc}}}{V_{\text{rms}}} = 2\sqrt{2} \left[ \frac{r}{2} \right]^{-0.5} \quad (6)$$

### 3. Comparison of special cases in Hernquist and plummer models

From the analysis of Narasimhan et al. (1997) and Ahmed (1979) we obtain when  $r = 0$

$$\begin{aligned} \left[ \frac{V_{\text{esc}}(0)}{(V_{\text{rms}})_1} \right]^2 &= 12(1 + M_{21}) H(\alpha_{12}) \quad \dots \text{Hernquist's model} \\ &= \frac{64}{3\Pi} (1 + M_{21}) A(\alpha_{12}) \quad \dots \text{Plummer model} \end{aligned} \quad (7)$$

where  $M_{21} = M_2 / M_1$

When  $\alpha_1 \gg \alpha_2$ ,  $\alpha_{21} \rightarrow 0$ ,  $H(\alpha_{12}) \rightarrow 1$ ,  $A(\alpha_{12}) \rightarrow 1$ . Hence  $[V_{\text{esc}}(0)/(V_{\text{rms}})_1]^2 = 12$  for Hernquist's model and  $64/3\Pi$  for plummer Model. Thus for both the models,  $V_{\text{esc}}(0)/(V_{\text{rms}})_1 \leq 2\sqrt{3}$  always. When  $\alpha_1 = \alpha_2$ ,  $\alpha_{12} = 1$ ,  $H(\alpha_{12}) = 1/3$ ,  $A(\alpha_{12}) = 3\Pi/16$ , we get  $V_{\text{esc}}(0)/(V_{\text{rms}})_1 = 2\sqrt{2}$ . This result holds good for polytropes also (Sastry and Alladin, 1981). Thus we should expect the inequality  $2\sqrt{2} \leq V_{\text{esc}}(0)/(V_{\text{rms}})_1 \leq 2\sqrt{3}$  to hold good for all galactic, models considered.

#### 4. Discussion

To facilitate comparison between the predictions of different models used to represent galaxies, it is convenient to keep the mass  $M$  and the self-gravitational potential energy  $\Omega$  (or the dynamical radius  $\bar{R}$ ) the same in all the models. It follows from Equation (3) that the  $V_{\text{rms}}$  then is independent of the model. Further, Narasimhan et al. (1997) have shown that  $W(r)$  obtained from Hernquist's model closely agrees with those obtained for plummer and polytropic models for the same mass and dynamical radius. Hence  $V_{\text{esc}}/V_{\text{rms}}$  is practically independent of galactic models considered. Sastry and Alladin (1981) had obtained numerically  $V_{\text{esc}}/V_{\text{rms}}$  for various polytropic models of identical galaxies and indicated that  $V_{\text{esc}}(r)$  expressed in units of  $V_{\text{rms}}$  was independent of the model if  $r$  was expressed in units of  $\bar{R}$ .

It may be noted that the simple formula given in (6) is a good approximation for  $s > 2$ . At  $s = 2$  the exact formula (5) gives  $V_{\text{esc}} / V_{\text{rms}} = 1.86$  and the approximate formula (6) gives 2.00.

The limiting velocity of escape will depend on the velocity distributions of stars in the galaxies and may not be independent of the galactic models. In fact,  $V_{\text{esc}}$  derived above is actually the lower limit to the actual value, since the transfer of energy from the orbital motion of the galaxies into the internal degrees of freedom during the collision between the galaxies has not been taken into account (Sastry and Alladin 1981). A more accurate work should also take into account the change in potential energy during the encounter as is done in N-body simulations.

#### References

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