POLARIZATION OF STARLIGHT BY UNRESOLVED AND OBLATE EXTRA-SOLAR PLANET IN ELLIPTICAL ORBIT

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ABSTRACT

We calculate the degree of linear polarization of radiation from stars having planets that may not be resolved spatially. We assume single scattering by water and silicate particulates in the planetary atmosphere. The dilution of the reflected polarized radiation of the planet by the unpolarized stellar radiation and the effect of oblateness of the planet as well as its elliptical orbit are included. We employ a chemical equilibrium model to estimate the number density of water and silicate condensates and calculate the degree of linear polarization at R band of starlight as a function of (1) mean size of condensates, (2) planetary oblateness, (3) inclination angle, (4) phase angle, (5) orbital eccentricity \(e\) and (6) the epoch of periastron passage. We show that the polarization profile alters significantly at all inclination angles when elliptical orbit is considered and the degree of polarization peaks at the epoch of periastron passage. We predict that detectable amount of linear polarization may arise if the planetary atmosphere is optically thin, the mean size of the condensates is not greater than a few microns and the oblateness of the planet is as high as that of Jupiter.

\textit{Subject headings:} binaries:general – dust,extinction – polarization – scattering – planetary systems
1. INTRODUCTION

Polarization has always been an efficient tool to probe the physical properties in the environment of various astrophysical objects. Recently, it is realized that polarization could be a very important diagnostic method for analyzing the atmosphere of sub-stellar massive object such as brown dwarfs and extra-solar planets. Sengupta & Krishan (2001) predicted detectable amount of linear polarization from L dwarfs because of the presence of condensates in the visible atmosphere and subsequently linear polarization at optical is detected by Menard, Delfosse & Monin (2002) and Zapatero Osorio et al. (2005). The observed linear polarization can well be explained by single dust scattering models that assumes optically thin and rotationally oblate photosphere (Sengupta 2003; Sengupta & Kwok 2005).

On the other hand, the use of polarimetry in detecting and understanding the physical properties of extra-solar planets is emphasized by Seager et al. (2000); Saar & Seager (2003); Stam et al. (2004). While Seager et al. (2000) modeled the polarization due to close-in-planet or the so called roaster such as the first discovered extra-solar planet 51 Peg b (Mayor & Queloz 1995), Stam et al. (2004) estimated the degree of polarization caused by planet at Jupiter's distance from the Sun. In both the methods, the reflected flux of the planet and the Stokes vectors of the reflected radiation are calculated by assuming circular orbit and blackbody radiation from the star. The planet is considered to be spherical and the polarization is integrated over the illuminated portion of the disk. While multiple scattering polarization is calculated by Stam et al. (2004), Seager et al. (2000) employed Monte-Carlo procedure and computed the number of scatterings. The later authors found that for an almost absorbing atmosphere, higher amount of polarization may arise due to single scattering. In both the investigations, the observable degree of polarization is estimated by simply multiplying the flux ratio with the polarization of the radiation from the spatially resolved planet. Since degree of polarization is a relative measure, it does not carry any information on the radius of the circular orbit or the planetary radius.

However, in the most realistic situation, the planet should be oblate because of rotation around its own axis. The oblateness of solar planets ranges from 0.003 for Earth to 0.065 for Jupiter and 0.1 for Saturn. Further, most of the extra-solar planets detected so far have eccentric orbit around the star and the eccentricity ranges from 0.0 to as high as 0.7.

Although multiple scattering is a reasonable choice for optically thick medium, it underestimates the amount of polarization by a few orders if the medium is optically thin and hence if polarization is caused by single scattering. As mentioned before, single scattering polarization could successfully explain the high degree of linear polarization observed from L dwarfs. Therefore, it’s quite reasonable to investigate the amount of polarization that could arise due to single scattering in extra-solar planets.
In this paper, we present polarization profiles of starlight caused by single scattering in the atmosphere of an oblate planet rotating in elliptical or bit and show that in certain cases the polarization may be detectable by the existing instrumental facilities.

2. SINGLE SCATTERING POLARIZATION OF STARLIGHT

Since the dust density is assumed to be low and scatterings by atoms and molecules (e.g., Rayleigh scattering) do not contribute to polarization significantly, single scattering approximation is reasonable for the region where the optical depth $\tau < 1$. If present, multiple scattering can reduce the degree of polarization by a few orders of magnitude (Sengupta & Krishan 2001) because the planes of the scattering events are randomly oriented and average each other’s contribution out from the final polarization. Hence, the amount of observed linear polarization could act as a probe in favour of single or multiple scattering approximation.

In the present work, we use the formalism given by Simmons (1983) which is a generalization of the work by Brown, McLean & Emslie (1978). In this formalism, the primary star is assumed to be a point source of unpolarized light. For most practical application, the Stokes parameter are normalized and defined by $[I_{\text{refl}}, Q, U, V]/I_{\text{total}}$ where $I_{\text{total}} = I_{\text{refl}} + I_{\text{star}}$, $I_{\text{refl}}$ being the reflected flux or intensity from the planet and $I_{\text{star}}$ being the unpolarized flux or intensity received from the star. We have neglected the thermal radiation of the planet if any because for a sufficiently old planet the thermal radiation should be much less as compared to the reflected radiation or there may not be thermal radiation by comparatively smaller, earth like planets. However, for young giant planets far away from the primary star, thermal radiation should be significant. Since $I_{\text{refl}}$ is much less than $I_{\text{star}}$, we consider $I_{\text{total}} = I_{\text{star}}$ and hence after we shall refer $I_{\text{refl}}, Q, U$ and $V$ to normalized Stokes parameter. Hence the degree of linear polarization can be written as $P = \sqrt{Q^2 + U^2}$.

The density distribution of scatterer is calculated in co-rotating reference frame so that it is independent of the phase angle $\Lambda$ of the planet. The transformation to observer frame is done by using the properties of spherical harmonics under rotation. In a circular orbit, the normalized Stokes parameter $Q$ and $U$ are given as a harmonic series:

$$Q(k, i, \Lambda) = \sum_{m=0}^{\infty} [p_m(k, i) \cos m\Lambda + q_m(k, i) \sin m\Lambda] \quad (1)$$

$$U(k, i, \Lambda) = \sum_{m=0}^{\infty} [u_m(k, i) \cos m\Lambda + v_m(k, i) \sin m\Lambda] \quad (2)$$
where \( k = 2\pi/\lambda, \) \( \lambda \) being the wavelength, \( i \) is the orbital inclination angle. The harmonic co-efficients are given by

\[
\begin{pmatrix} p_m \\ q_m \end{pmatrix} = \frac{2\pi}{k^2} \sum_{l=M}^{\infty} F_{l2}(k) G_m^l(i) \left( \frac{\eta_{lm}}{\xi_{lm}} \right), m = 0, 1, 2, 3, \ldots \tag{3}
\]

\[
\begin{pmatrix} u_m \\ v_m \end{pmatrix} = \frac{2\pi}{k^2} \sum_{l=M}^{\infty} F_{l2}(k) H_m^l(i) \left( \frac{-\xi_{lm}}{\eta_{lm}} \right), m = 0, 1, 2, 3, \ldots \tag{4}
\]

\[ M = \max(2, m) \] and \( G_m^l(i), H_m^l(i) \) are given in Simmons (1983). \( \eta_{lm} \) and \( \xi_{lm} \) are related with the density distribution in the co-rotating frame and are given by

\[
\begin{pmatrix} \eta_{lm} \\ \xi_{lm} \end{pmatrix} = \alpha(l, m) \int n'(r, \theta, \phi) P_l^m(\cos \theta) \left( \frac{\cos m\phi}{\sin m\phi} \right) \sin \theta d\theta d\phi dr. \tag{5}
\]

where \( n'(r, \theta, \phi) \) is the number density of scatterer in the co-rotating frame, \( \theta_i \) is the viewing angle, \( P_l^m \) is the associated Legendre function of the first kind and

\[
\alpha(l, m) = \left[ \frac{(2l+1)(l-m)}{4\pi(l+m)} \right]^{1/2}. \tag{6}
\]

At an edge-on view, \( \theta_i = \pi/2 \) and \( \phi = 0 \), and hence \( \xi_{lm} = 0 \). \( F_{l2}(k) \) is related to the scattering function and is given by

\[
F_{lm} = \alpha(l, m) \int_{-1}^{1} \frac{i_1(k, \theta) - i_2(k, \theta)}{2} P_l^m(\cos \theta) d(\cos \theta). \tag{7}
\]

In the above equation, \( \theta \) is the scattering angle, \( i_1 \) and \( i_2 \) are the scattering functions given by van de Hulst (1957). \( i_1 \) and \( i_2 \) depend on the refractive index as well as on the size and shape of the scatterer. In the present work we consider spherical dust particles as scatterer.

Considering an ellipsoidal density distribution and using the addition theorem of spherical harmonic, \( \eta_{lm} \) can be written as

\[
\eta_{lm} = 2\pi \alpha(l, m) P_l^m(0) \int_{R_1}^{R_2} n(r) dr \int_{-1}^{1} \frac{P_l(\mu) d\mu}{[1 + (A^2 - 1)\mu^2]^{1/2}}. \tag{8}
\]

where \( R_1 \) and \( R_2 \) are the outer and the inner equatorial axis length of the planet, \( A \) is the ratio of the length of the equatorial axis to the polar axis, and \( \mu = \cos \theta \). We have taken multi-poles up to \( l=5 \). We have considered up to fifth harmonic, i.e., \( m=0,1,2,3,4,5 \). However, since the density distribution is symmetric about the orbital plane, the first, the third and the fifth harmonics are zero and the fourth harmonic is small as compared to the second
harmonic. In other word, for the adopted density distribution, the odd number harmonics are zero and the degree of polarization is determined mainly by the second harmonic.

We convert $n(r)dr$ into $n(P)dP$ by using the equation of hydrostatic equilibrium $n(r)dr = n(P)dP/g\rho(P)$, where $P$ is the pressure at different geometrical height, $\rho$ is the mass density at different pressure scale, and $g$ is the surface gravity (which can be assumed to be constant for a geometrically thin atmosphere). The degree of polarization is calculated at wavelengths ranging from 0.5486 to 0.8491 $\mu$m and averaged by using the response function of R band Bessel filter.

3. EFFECT OF ORBITAL ECCENTRICITY

The effect of orbital eccentricity enters through a multiplicative factor $h(\Lambda, e)$ to the co-efficient of the dominant harmonic $p_2$ and $v_2$ (Brown et al. 1982) and it is given by

$$h(\Lambda, e) = \frac{[1 + e \cos(\lambda - \lambda_p)]^2}{(1 - e^2)^2}$$

where $e$ is the orbital eccentricity and $\lambda$ is the true anomaly which is related to the eccentric anomaly $E$ by the relationship

$$\tan \left(\frac{\lambda - \lambda_p}{2}\right) = \left(\frac{1 + e}{1 - e}\right)^{1/2} \tan \frac{E}{2},$$

and $\lambda_p$ is the longitude of the periastron. The eccentric anomaly $E$ is related to the orbital phase angle through Kepler’s equation

$$E - e \sin E = \Lambda - \Lambda_p$$

where $\Lambda - \Lambda_p = 2\pi(t - T_0)/P$ is the mean anomaly, $t$ being any epoch of time, $P$ is the orbital period and $T_0$ is the epoch of periastron passage. We have assumed no drastic seasonal change in the planetary atmosphere and that the atmospheric T-P profile remains the same through out the planetary year.

For a circular orbit, the polarization profile is determined by the second harmonic only. But for an elliptical orbit, the first and the third harmonics are non-zero although the second harmonic remains dominant.

4. THE ATMOSPHERIC MODELS

We have adopted the atmospheric models of extra-solar planets given in Sudarsky et al. (2003). The Temperature-Pressure (T-P) profiles are taken for models of a class II and a
class V planets or closed-in-planet also known as roaster. The atmosphere of class II planets should have water as condensates while class V planets should contain silicate in their upper atmosphere. In calculating the T-P profile of the planet Ups And d (a class II planet), Sudarsky et al. (2003) considered the surface gravity to be $2 \times 10^4$ cm$^{-2}$ while for the planet HD 209458 b (a class V planet) the surface gravity is taken to be 980 cm$^{-2}$. In the present work, we have taken the same values for the surface gravity of the two representative planets. We have assumed that the thermal radiation of the planet is negligible compared to the reflected radiation so that the contribution to polarization comes only from the reflected radiation.

The dust distribution in the atmosphere is calculated based on the one-dimensional cloud model of Cooper et al. (2003). This model assumes chemical equilibrium throughout the atmosphere, and uniform density distribution across the surface of an object at each given pressure and temperature. Under these assumptions, the number density of cloud particles is given by

$$n(P) = q_c \left( \frac{\rho}{\rho_d} \right) \left( \frac{\mu_d}{\mu} \right) \left( \frac{3}{4\pi r^3} \right),$$

where $\rho$ is the mass density of the surrounding gas, $r$ is the cloud particle radius, $\rho_d$ is the mass density of the dust condensates, $\mu$ and $\mu_d$ are the mean molecular weight of atmospheric gas and condensates respectively. The condensate mixing number ratio ($q_c$) is given as $q_c = q_{\text{below}} P_{c,l} / P$ for heterogeneously condensing clouds where $q_{\text{below}}$ is the fraction of condensible vapor just below the cloud base, $P_{c,l}$ is the pressure at the condensation point, and $P$ is the gas pressure in the atmosphere. The condensation curves for water and forsterite condensates are taken from Sudarsky et al. (2003). The values of $\mu_d$, $\rho_d$ and $q_{\text{below}}$ for forsterite and water are taken from Cooper et al. (2003). The real part of the refractive index for forsterite is fixed at 1.65 at any wavelength and the imaginary part is taken by interpolating the data given in Scott & Duley (1996). For the refractive index of water, we have used the data given in Segelstein (1981).

Apart from the calculation of the grain number density, the location of the cloud in the atmosphere plays an important role in determining the amount of polarization. The location of the cloud base for different atmospheric models and different chemical species is determined by the intersection of the T-P profile of the atmosphere model and the condensation curve $P_{c,l}$ as prescribed in Cooper et al. (2003). Taking the condensation curve for forsterite and water as given in Sudarsky et al. (2003), we determine the base of the cloud from the T-P profiles of Ups And d and HD209458 b calculated by the same authors.

At present, there is no convincing justification in favor of any specific form of the particle size distribution function. In the present work, we adopt a log-normal size distribution
function used by Ackerman & Marley (2001) and Saumon et al. (2000) with a fixed width at 1.3. Following Sudarsky et al. (2003), we have taken the mean particle radius for water 5.0 $\mu m$ and that for silicate (forsterite) 10.0 $\mu m$. However, in order to show the effect of mean grain size on the degree of polarization we have also presented polarization profile as a function of mean particle size keeping the T-P profile the same as that calculated by Sudarsky et al. (2003) for water with mean grain size 5.0 $\mu m$.

In multiple scattering, the mean particle size should be different at different pressure scale. However, for single scattering, a fixed mean particle size of a particular condensate is sufficient (Sengupta & Kwok 2005).

5. THE OBLATENESS OF PLANETS

The rotationally induced oblateness of solar planets has been discussed in details by Hubbard (1984) and Murray & Dermott (2000). Recently, the formalism for oblateness is extended to extra-solar planets by Barnes & Fortney (2003) who used Darwin-Radau relationship

$$f = \frac{\Omega^2 R_e^3}{GM} \left[ \frac{5}{2} \left( 1 - \frac{3}{2} K \right)^2 + \frac{2}{5} \right]^{-1}$$

(Murray & Dermott 2000; Barnes & Fortney 2003) to relate rotation to oblateness. In the above equation, $K = I/MR_e^2 \leq 2/3$ is the moment of inertia parameter of an object with moment of inertia $I$. The Darwin-Radau relationship is exact for uniform density objects ($K = 0.4$) and provides a reasonable (within a few percent of errors) estimation of the oblateness of the solar planets. At 1 bar pressure level, the oblateness $f$ of Jupiter, Saturn and Earth are 0.065, 0.098 and 0.003 respectively. Barnes & Fortney (2003) modeled the planet HD209458b and estimated its oblateness to be about 0.00285. Apart from rotational effects, tidal interaction of a closed in planet with its primary star may also impose an ellipsoidal shape extending toward the star. In the present work we ignore such effects. Moreover, the estimation of the moment of inertia of the planets needs density distribution, radius and mass of the planet along with its rotational period and so it is highly model dependent. In the present work we adopt a wide range of values for oblateness, the maximum value is equal to that of Jupiter ($f = 0.065$) and the minimum value is equal to that of Earth($f = 0.003$).
6. RESULTS AND DISCUSSION

We present in figure 1 and in figure 2, the polarization profiles at R band (Bessel filter) of star with a planet having forsterite as a major condensate in its atmosphere. Silicate and iron can form high in the atmosphere of class V roosters that orbit their stars around 0.05 AU. Although presence of more than one species affects the amount of polarization because of differences in size, optical properties, and location, we assume forsterite as the major condensate in our model for simplicity. However, the change in the amount of polarization by the incorporation of other condensate species cannot be determined due to the dominant effect of other parameters such as inclination angle, orbital eccentricity, oblateness etc. Figure 3 presents the degree of polarization as a function of planetary oblateness. Figure 4 presents the polarization profile for planets with non-circular orbit and the change in degree of polarization caused by the change in orbital eccentricity is shown in figure 5.

Our polarization profiles for the orbital eccentricity $e=0$ i.e., for circular orbit, presented in figure 1 and figure 2, are qualitatively the same as that presented by Stam et al. (2004). When the inclination angle $i = 0$, the observed part of the planet that is illuminated is constant and hence the degree of polarization is constant. The maximum amount of polarization produced by the planet is the same for all values of $i$. When the inclination angle $i = 90^\circ$, the polarization is zero at the fractional orbital period $t/P = 0.25$ and 0.75 that correspond to the phase angle $\Lambda = 180^\circ$ and $0^\circ$ respectively because the planet’s night and day side are turned toward the observer in these cases. Apart from the geometrical asymmetry of any object, another asymmetry plays important role for a binary star or a planet around a star in determining the polarization profile. This second effect is due to the asymmetry in the position of the object (or the scatterer) with respect to the observer and the source of light. As the symmetry axis of the planet changes position with respect to the observer, the total number of effective scatterers also changes. This effect can be visualized as the amount of light that is scattered towards the observer from the net illuminated region. In a circular orbit, the observed degree of polarization is determined by the scatterers that are present within the illuminated area facing towards the observer. The observed degree of polarization is zero when the night side of the object is towards the observer and it increases as the illuminated area towards the observer increases. The fraction of the area that is illuminated depends on the inclination angle as well. As the inclination angle decreases, the orbital plane becomes more and more face on towards the observer. Consequently, the illuminated area towards the observer increases yielding into higher amount of polarization that can be seen by the observer.

However, the polarization profile alters drastically when the orbit is non-circular as can be seen in figure 4. The degree of polarization peaks when the planet passes its periastron.
Hence, the degree of polarization is time variable even if the inclination angle is zero degree. The change in the amount of polarization is determined by the eccentricity of the orbit as can be seen from Figure 5.

As discussed by Brown et al (1982), the "weighted" or effective optical depth (as seen by the source) of the scattering region idealized as point scatterer containing N particles is proportional to the square of the ratio between the semi-major axis $a$ and the stellar separation at longitude $\lambda$ given by $R(\lambda) = a(1-e^2)/(1 + e \cos(\lambda - \lambda_p))$. For a circular orbit this ratio is identically 1 but for elliptical orbit the ratio is $[1 + e \cos(\lambda - \lambda_p)]/(1 - e^2)$. This means, for an elliptical orbit the effective optical depth of the scattering region as seen by the source is increased by a factor of $h(\lambda, e)$ given in section 3. For a circular orbit, $e=0$ and the effective optical depth and hence the degree of polarization becomes independent of the longitude. When $\lambda = \lambda_p$, i.e., when the planet is at the periastron, the effective optical depth as seen by the source is the maximum yielding maximum amount of polarization.

As shown by Sengupta & Krishan (2001); Sengupta (2003); Sengupta & Kwok (2005), the oblateness of the atmosphere plays a crucial role in determining the degree of polarization. The net degree of polarization is calculated by integrating the polarization at all points over the planetary disk. Since, we take the number density of scatterers distributed about some symmetry axis, the positive part of polarization gets cancelled out by the negative part. Now as oblateness increases, the net cancellation decreases owing to the departure from spherical symmetry. Consequently, the disc integrated polarization would vanish if the geometry is perfectly spherical and departure from sphericity would give rise to non-zero polarization. An increase in oblateness means more asymmetry and hence more net non-zero polarization. Figure 1 shows that if the oblateness of the planet is as small as 0.003 (similar to that of Earth) then the degree of polarization is of the order of $10^{-4}$ for the mean grain size of forsterite 10.0 $\mu m$. The degree of polarization estimated by Seager et al. (2000) for similar planets is of the order $10^{-6}$ for grain size 1-10 $\mu m$. This is because of the fact that multiple scattering reduces the amount of polarization by a few orders of magnitude as discussed in section 2. However, if the oblateness of the object is increased, the degree of polarization increases substantially as shown in figure 2 and could be at the range of present detectibility. In figure 3 we present the degree of polarization by forsterite grain of mean radius 10 $\mu m$ for different values of the planetary oblateness.

Figure 6 shows the polarization profiles of star with class II planet that orbits at a distance of 1-2 A.U and hence should have atmospheric temperature about 250 K. This type of planet should have a tropospheric cloud layer of water along with lesser amount of methane and ammonia.

The amount of polarization is although dependent on the optical property of the con-
densates, the size of the condensates play the dominant role in determining the amount of polarization. According to the adopted chemical equilibrium model by Cooper et al. (2003), the particle number density increases with the decrease in grain size. As a result, the amount of polarization increases substantially when the mean particle size is decreased. Therefore, high degree of polarization might arise in a less oblate planet if the grain size is sufficiently small. On the other hand, large amount of polarization can be produced if the grain size is large but the oblateness is high as shown in figure 6. In figure 7, we present the degree of polarization caused by water particulates with different mean size. However, in order to understand the effect of grain size, we have kept the T-P profile the same as that calculated by Sudarsky et al. (2003) with mean water particulate size 5 µm.

It may not be possible to infer from the polarization profile, the specific nature of the condensates, the oblateness of the planet or the orbital inclination angle as the amount of polarization would arise by a combination of all these parameters. However, if the distance of the planet from the primary star is known, then the nature of the condensate can be determined. Exact understanding on the chemical equilibrium may provide the size of the condensates. As a result, a combination of the oblateness and the inclination angle of the planet can be determined from the polarization data which in turn can provide information on the mass and the rotational velocity of the planet. However, the periastron angle and the orbital eccentricity can be inferred, irrespective of the amount of polarization, from the phase angles at which the maximum and the minimum amount of polarization arise.

7. CONCLUSION

The important message that is conveyed in this paper is that single scattering in a rotationally induced oblate planet may cause sufficient amount of linear polarization that could be detected by using the available instrumental facilities. The amount of linear polarization would be sufficiently large if the planet is highly oblate and the mean size of the condensates is not greater than a few micron. If detected, the polarization profile will provide information on the oblateness and orbital inclination of the planet and hence on the spin period and mass of the planet. Further, the time dependent polarization profile would provide the orbital period and the eccentricity. Our present discussion is confined to stars with single planet. The case for more than one planet orbiting a star at different inclination and phase angle is worth investigating as that should change the amount and the periodic nature of the variability of the polarization. The present investigation is aimed for the polarimetry of already known extra-solar planets. Since, the orbital separation and the emitted flux from the parent star are known, one can employ the theoretical inferences about the composition
and location of the condensates in the planetary atmosphere. However, in general, the degree of polarization should depend on the evolution of the planets. Baraffe et al. (2003) have discussed on the time evolution of extra-solar planets and showed how the radius and hence the surface gravity as well as the effective temperature of an irradiated planet changes with time. Since the formation, chemical evolution and the geometrical location of condensates change with the change in surface gravity and atmospheric temperature profile, the degree of polarization, whether by single scattering or by multiple scattering, would also alter with time. Therefore, time evolution of the polarization profile is worth investigating.

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Fig. 1.— Degree of linear polarization at Bessel R band caused by forsterite grain with mean radius $r_0 = 10 \mu m$. The polarization is plotted as a function of the fractional orbital period $t/P$ where $t$ is time and $P$ is the orbital period. Models for planet with oblateness $f=0.003$ and circular orbit. From bottom to top at $t/P = 0.25$, the curves represent polarization with inclination angles $i = 90^\circ$, $60^\circ$, $45^\circ$, $30^\circ$ and $0^\circ$ respectively.
Fig. 2.— Same as figure 1 but for planet with oblateness $f=0.01$. 
Fig. 3.— Degree of polarization at Bessel R band as a function of oblateness $f$. Circular orbit with inclination angle $i = 90^\circ$. The linear polarization by planet with forsterite with $r_0 = 10\mu m$ is calculated at the fractional orbital period $t/P = 0.0$. 
Fig. 4.— Same as figure 1 but for planets orbiting in elliptical orbit with eccentricity $e = 0.3$ and the epoch of periastron passage $T_0/P = 0$. 
Fig. 5.— Degree of polarization by planet with $f=0.065$ and $T_0/P = 90^\circ$ but different orbital eccentricity. $t/P$ is the fractional orbital period, $t$ being time and $P$ the orbital period. Solid lines from bottom to top at $t/P = 0.5$ represent models with $e = 0.0, 0.1, 0.2, 0.3$, and $0.4$ respectively and with $i = 90^\circ$ while the dashed lines represent that with $i = 30^\circ$. 
Fig. 6.— Degree of polarization as a function of fractional orbital period $t/P$ at Bessel R band due to water condensates. (a) Models for planet with $f=0.003$, $e=0.4$ and mean particle radius $r_0 = 5.0\mu m$. Solid lines - models with $\Lambda_P = 60^\circ \ (T_0/P = 0.67)$; dashed lines - models with $\Lambda_P = 120^\circ \ (T_0/P = 0.42)$. Both the solid and the dashed lines from bottom to top at $t/P = 0.25$ represent polarization with $i = 90^\circ$, 60$^\circ$, 45$^\circ$, 30$^\circ$ and 0$^\circ$ respectively. (b) Same as (a) but solid lines - models with $r_0 = 1.0\mu m$, $f=0.001$ and $T_0/P = 0.25$; dashed lines - models with $r_0 = 4.0\mu m$, $f=0.01$ and $T_0/P = 0.75$. 
Fig. 7.— Degree of polarization at Bessel R band as a function of mean grain diameter $d_0$. Circular orbit with inclination angle $i = 90^\circ$. The linear polarization by planet with oblateness $f=0.001$ having water is calculated at $t/P = 0.0$. 

Water
$t/P=0.0$
$f=0.001$