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## Fast Numerical Methods for Polarized Line Radiative Transfer in the Presence of Hanle Effect

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**Abstract.** The Hanle effect provides a diagnostic tool for weak magnetic fields which do not give rise to a measurable Zeeman effect, such as turbulent fields or magnetic canopies. The lines which are sensitive to the Hanle effect are formed under non-LTE conditions, by scattering of photons.

Inversion methods for such diagnostics require to solve the non-LTE polarized transfer equation for a large number of magnetic configurations. Fast numerical methods are thus highly required. We present an Approximate Lambda Iteration method to treat the Hanle effect for lines formed with complete frequency redistribution. Referred to as PALI-H, this method is an extension of ALI methods first developed for non polarized line transfer. The starting point is to recast the polarized transfer equation into a vectorial integral equation for a 6-component source function. We show that the convergence of the method is independent of the strength and direction of the magnetic field. The method is very fast and allows to handle any type of depth-dependent magnetic field.

### 1. Introduction

Diagnostic tools for weak magnetic fields are needed for the study of non-active regions in the solar photosphere or chromosphere. In the photosphere, there is now some evidence that weak fields do exist outside magnetic flux-tubes, on the form of mixed polarity fields (Faurobert-Scholl et al. 1995, Stenflo & Keller 1997). In the quiet chromosphere the expansion of flux-tubes gives rise to weak almost horizontal magnetic fields forming a magnetic canopy. Those magnetic configurations are very difficult to detect via the Zeeman effect because of the magnetic flux cancellation at small scale. But they may be detected via their Hanle effect.

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Let us recall briefly how the Hanle effect works. Scattering of photons in a spectral line leads to linear polarization of the reemitted radiation field if the incident radiation is anisotropic. The physical origin is the quantum counterpart of Rayleigh scattering by small particules. Resonance polarization is observed in the solar spectrum in a large number of lines, close to the solar limb (Stenflo and Keller 1997). Because of the axial symmetry of the photospheric incident radiation, the polarization is zero at disk center and it increases steeply towards the limb. The limb-darkening of the photospheric light leads to polarization in the direction parallel to the solar limb.

In the presence of a weak magnetic field, when the Zeeman splitting of the atomic levels is on the order of their inverse life-time, the linear polarization of the scattered radiation is rotated with respect to its non-magnetic direction and the polarization rate is reduced. This is the Hanle effect. It has been observed in several lines formed in the low chromosphere, such as the CaI 4227 Å and the SrII 4078 Å resonance lines (Bianda et al. 1998). It may be used for the diagnostics of magnetic canopies in the chromosphere. In the presence of a weak magnetic field with mixed polarity as small scale, there is no net rotation of the polarization plane but the depolarization is not cancelled out. It may be used as a diagnostic tool for turbulent magnetic fields.

Such diagnostics require to develop inversion codes based on the solution of the radiative transfer equation for a polarized line formed in the presence of Hanle effect. Fast numerical methods are thus needed to solve this equation. In the following part we show how this problem may be cast into an integral equation for a reduced source function. In the third part, we present an accelerated lambda iteration technic for solving this integral equation.

## 2. Polarized transfer equation and reduced problem

We give here the main steps of the derivation of the integral equation, for a detailed description see Nagendra et al., 1998. We consider a linearly polarized line formed with complete frequency redistribution in a plane parallel atmosphere with a given weak magnetic field.

In the Stokes parameter formalism the radiation field is described by a 3-component vector  $\vec{I} = (I, Q, U)^\dagger$ . Because of the presence of the magnetic field the radiation field is not axially-symmetrical, it depends on the 4 variables  $(\tau, \nu, \mu, \varphi)$ , as usual  $\mu = \cos \theta$  where  $\theta$  is the colatitude of the line of sight,  $\varphi$  is its azimuth,  $\tau$  and  $\nu$  are respectively the line optical depth and the frequency.

One can show that in the presence of a weak magnetic field the absorption matrix is scalar, and that the absorption coefficient is the same as in the non-magnetic case. The transfer equation for  $\vec{I}$  is

$$\mu \frac{d\vec{I}}{d\tau} = (\phi(\nu) + \beta)(\vec{I} - \vec{S}), \quad (1)$$

where  $\phi$  is the absorption profile in the line and  $\beta$  is the ratio of the line to the continuous absorption coefficients,  $\vec{S}$  is a 3-component source vector which is the sum of a scattering term and of a thermal creation term

$$\vec{S}(\tau, \mu, \varphi) = (1 - \varepsilon) \int \phi(\nu) d\nu \int \frac{d\Omega'}{4\pi} \hat{P}_B(\mu, \varphi, \mu', \varphi') \vec{I}(\tau, \nu, \mu', \varphi') + \varepsilon \vec{B}, \quad (2)$$

where  $\hat{P}_B$  is the Hanle phase matrix and  $B$  is the Planck function.

In order to reduce the problem we first perform an azimuthal Fourier expansion of the source vector. The same technic was used by Chandrasekhar for the problem of Rayleigh scattering in a planetary atmosphere. One can show that, because of the form of the Hanle phase matrix, the Fourier expansion has no term of order larger than 2, i.e.,

$$\vec{S}(\tau, \mu, \varphi) = \vec{S}_0(\tau, \mu) + \sum_{k=1}^{k=2} [\vec{S}_k(\tau, \mu) \cos k\varphi + \vec{S}_{-k}(\tau, \mu) \sin k\varphi]. \quad (3)$$

Furthermore, it is possible to factorize the  $\tau$  and  $\mu$  dependence of the Fourier components. Let us introduce the Fourier vector

$$\vec{S}_F = (\vec{S}_0, \vec{S}_1, \vec{S}_{-1}, \vec{S}_2, \vec{S}_{-2})^\dagger. \quad (4)$$

One can show that

$$\vec{S}_F(\tau, \mu) = \hat{B}(\mu) \vec{S}_r(\tau), \quad (5)$$

where  $\vec{S}_r(\tau)$  is a 6-component vector depending on  $\tau$  only and  $\hat{B}$  is a matrix. The first component  $S_I$  is the same as for non-polarized lines.

The reduced source vector obeys an integral equation given by

$$\vec{S}_r(\tau) = (1 - \varepsilon) \hat{H}_B(\tau) \int_0^{+\infty} \hat{K}(\tau - \tau') \vec{S}_r(\tau') d\tau' + \varepsilon \vec{B}, \quad (6)$$

with

$$\vec{B} = (B, 0, 0, 0, 0, 0)^\dagger. \quad (7)$$

$\hat{K}$  is a 6X6 matrix kernel independent of the magnetic field on the form

$$\hat{K}(\tau) = \begin{pmatrix} K_{11} & K_{12} & 0 & 0 & 0 & 0 \\ K_{12} & K_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{44} \end{pmatrix}, \quad (8)$$

$K_{11}$  is the kernel which appears in the integral equation for non-polarized lines.  $\hat{H}_B$  is a 6x6 matrix depending on  $\vec{B}$  ( $\hat{H}_B$  is the identity matrix when  $\vec{B} = \vec{0}$ ). Let us remark here that the Hanle effect of the magnetic field appears in the integral equation as a *local* multiplication by the matrix  $\hat{H}_B$ .

### 3. Accelerated Lambda iteration method

The integral equation (6) for source vector  $\vec{S}_r$  may be solved by an iterative method of the approximate Lambda iteration type. This method is here briefly described, more details may be found in Nagendra et al. (1998) and in Faurobert-Scholl et al. (1997). We can write the integral equation in the symbolic form

$$\vec{S}_r(\tau) = (1 - \varepsilon)\hat{H}_B(\tau)\Lambda(\vec{S}_r) + \varepsilon\vec{B}. \quad (9)$$

This defines the Lambda operator which relates the source vector at one depth point to its values at any depth point in the medium. Its properties are derived from those of the (6x6) kernel matrix given in Eq. (8). We can define a generalized mean intensity as

$$\vec{J} = \Lambda(\vec{S}_r). \quad (10)$$

In order to solve Eq. (9) numerically, we first discretize the optical depth variable on  $N_T$  grid points. The discretized Lambda operator is then a ( $6N_T \times 6N_T$ ) matrix operating on the  $6N_T$  component source vector. Equation (9) is transformed into the linear system

$$[\hat{I}_d - (1 - \varepsilon)\hat{H}_B\Lambda](\vec{S}_r) = \varepsilon\vec{B}, \quad (11)$$

where  $\hat{I}_d$  is the ( $6N_T \times 6N_T$ ) identity matrix. For simplicity reasons we use the same notations for the “physical” quantities, such as the source vector, and for the discrete  $6N_T$  vectors. We denote by  $\hat{L}$  the ( $6N_T \times 6N_T$ ) operator on the left hand side of Eq. (11).

We have checked that this linear system of equations may be solved by an iterative method of the so-called “block-Jacobi” type. Assume that we know an estimate of  $\vec{S}_r$ , denoted by  $\vec{S}_r^{(n)}$ , the correction term  $\delta\vec{S}_r$  obeys the linear system

$$\hat{L}(\delta\vec{S}_r) = \varepsilon\vec{B} - \hat{L}(\vec{S}_r^{(n)}), \quad (12)$$

the right hand term is now the error term. Let us denote by  $\vec{J}^{(n)}$  the current estimate of the mean intensity, the error term is written as  $\varepsilon\vec{B} - (1 - \varepsilon)\hat{H}_B\vec{J}^{(n)}$ .

The basic idea of the method is to replace, for the calculation of the correction  $\delta\vec{S}_r$ , the full operator  $\hat{L}$  by an approximate operator  $\hat{L}^*$  which is easier to invert, and to iterate until the correction term gets small enough to reach the required accuracy for  $\vec{S}_r$ . At each iteration step the error term is computed accurately. The mean intensity is derived through a formal solution of the transfer equation with the current estimate of the source vector. Here  $\hat{L}^*$  is constructed by keeping only the (6x6) matrices  $\Lambda_{i,i}$  at each depth point  $i$ , this is the generalization of the “local approximation” introduced by Olson et al. (1986) for non-polarized transfer problems. For non-polarized problems the approximate operator is purely diagonal whereas in our polarized problem it is a block-diagonal operator, with (6x6) blocks on the diagonal. The correction to the source function is then obtained at each depth point by inverting a (6x6) matrix. The method for the computation of  $\hat{L}^*$  is described in Nagendra et al. (1998).

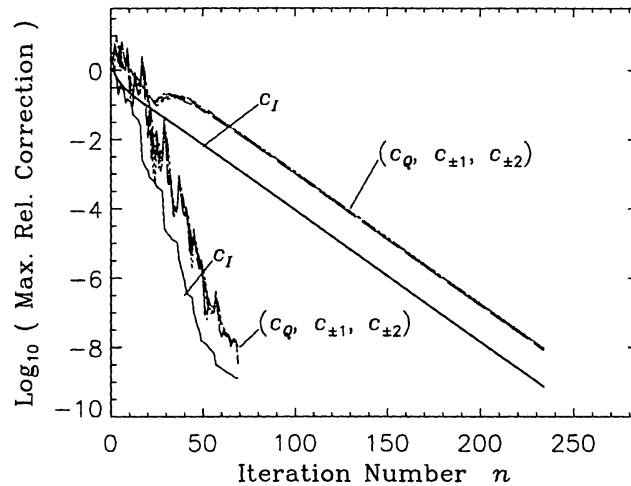


Figure 1. Decrease of the relative corrections on the 6 components of the source vector, without and with Ng acceleration. The medium is an isothermal slab of optical thickness  $T = 2 \cdot 10^9$ , destruction probability  $\varepsilon = 10^{-6}$ . The Voigt parameter of the line absorption profile is  $a = 10^{-3}$ . The direction of the magnetic field is given by  $\theta_B = 30^\circ$ ,  $\varphi_B = 0$ ; the Larmor frequency in the field, in units of the inverse life time of the atomic levels, is given by  $\gamma_B = 1$ .

We show on Fig. (1) the decreasing of the correction terms on the six components of the source vector, for an optically thick isothermal slab. The parameters for the magnetic field and the slab are given in the caption of the Figure. The correction term on the first component  $S_I$  behaves essentially as for non-polarized problems, the corrections on the polarized components show the same decreasing rate for large iteration numbers but they stay one order of magnitude larger. We recall that the convergence rate is controlled by the first component and depends essentially on the number of depth points per decade in the optical depth grid (see Faurobert-Scholl et al. 1997, and Olson et al. 1986). One important point here is that we have checked that it does not depend on the parameters of the magnetic field. As in non-polarized problems the convergence may be accelerated by an Ng acceleration technique.

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