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Radiative Transfer Effects and Convective Collapse: Size(flux)-Strength Distribution for the Small-Scale Solar Magnetic Flux Tubes

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Abstract. The effects of radiative energy exchange on the convective instability of a weak field magnetic structure, which lead to a prediction and a physical explanation of the magnetic flux dependent field strength, are examined in detail using a real model stratification for the photospheric and convection zone structure of the Sun. Adopting the generalised Eddington approximation for the radiative transfer, which is valid both in the optically thick and the thin limits, we model the lateral radiative energy exchange by the tube with the external medium with a self-consistent inclusion of vertical radiative losses.

1. Introduction

One of the important properties of the small-scale solar magnetic structures, as established by the observations (Stenflo and Harvey 1985, Zayer et al. 1989,1990, Schüssler 1991), is that while the strong field *network elements* have field strengths very weakly dependent on the flux per element, Φ, with typical values of Φ about 1×10^{18} Mx and higher, the *inner network* weak field structures have a typical value of about 500 G (Keller et al. 1994, Solanki et al. 1996, Lin 1995) for their strength and Φ about 1×10^{17} Mx or lower with a strong dependence between the two. If the convective collapse of a weak field tube is the global process responsible for the formation of the strong field tubes (Parker 1978, Webb and Roberts 1978, Spruit and Zweibel 1979, Spruit 1979, Hasan 1983, 1984,Venkatakrishnan 1985, Steiner 1996) that comprise the *network*, with strengths weakly dependent on Φ, then it should be explained why tubes with smaller fluxes, viz., the *inner network* elements, do not collapse to kG strength. Efficient radiative exchange with the surroundings by a small flux tube (Hasan 1986, Venkatakrishnan 1986) offers a natural explanation. Here, having in mind a quantitative comparison with the above mentioned observationally established properties of the solar flux tubes, we employ a semi-empirical model of the photospheric and the convection zone structure of the Sun and study in detail the effects of radiation on the convective instability and the wave motions.

2. Equations

We add to the familiar thin flux tube equations (Roberts & Webb 1978) the following non-adiabatic energy equation(see e.g., Cox 1980),

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial z} - \frac{\Gamma_1 p}{\rho}\left[\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial z}\right] = -\frac{\chi_T}{\rho c_v T}\nabla.\mathbf{F}, \tag{1}$$

where p, ρ, and $\mathbf{F}$ denote the fluid pressure, density, and radiative energy flux, respectively, (see Cox (1980) for the definitions of other thermodynamic quantities); all the variables are evaluated on the tube axis (r=0). The radiative flux $\mathbf{F}$ is calculated in the generalised Eddington approximation following Unno and Spiegel (1967): the mean intensity J that is needed to evaluate the flux,

$$\mathbf{F}_R = -\frac{4}{3\kappa\rho}\nabla J \tag{2}$$

, is found by reducing the exact relation

$$\nabla.\mathbf{F}_R = 4\kappa\rho(S - J) \tag{3}$$

to a form appropriate for a thin tube, which reads

$$\frac{1}{3}\frac{\partial^2 J}{\partial \tau^2} + \frac{4}{3}\left(\frac{J_e - J}{\tau_a^2}\right) = J - S, \tag{4}$$

where $d\tau = \kappa\rho dz$, $\tau_a = \kappa\rho a$, S is the source function which we take it to be the Planck function, a is the tube radius, and J_e is the mean intensity in the external medium. J_e is found by solving the equation

$$\frac{1}{3}\frac{\partial^2 J_e}{\partial \tau_e^2} = J_e - S_e. \tag{5}$$

2.1. Equilibrium

The equilibrium stratification of the external medium that we use here is the one determined to match the combined $VAL - C$ (Vernazza et al. 1981) and Spruit (1977) models (see Hasan, Kneer and Kalkofen (1998) for details about the construction of this external quite Sun model). The Rosseland mean opacities are calculated by interpolation from the tables of Kurucz (1993) for the upper layers and from those of Rogers and Iglesias (1992) for the deeper regions. We assume temperature equilibrium, $T = T_e$, which implies that β, defined as $\beta = 8\pi p/B^2$, is constant with z (if the dependence of μ is neglected). The pressure and density are thus determined from

$$p = \frac{\beta}{1+\beta}p_e \tag{6}$$

$$\rho = \frac{\beta}{1+\beta}\rho_e. \tag{7}$$

The extent of the flux tube covers the atmosphere from the temperature minimum in the chromosphere to 5000km deep in the convection zone with the photospheric surface ($\tau = 1$) being assigned $z = 0$. We measure the positive z downwards.

2.2. Linear stability: the perturbation equations

With the assumption that the perturbations in the ambient medium are negligible, small perturbations inside the tube about the equilibrium described above obey the following equations in the linear limit:

$$\frac{\partial \xi}{\partial z} = \frac{B'}{B} - \frac{\rho'}{\rho} - \left[\frac{d(\ln \rho)}{dz} - \frac{d(\ln B)}{dz}\right]\xi \tag{8}$$

$$\frac{\partial^2 \xi}{\partial t^2} = -Hg\frac{\partial}{\partial z}\left(\frac{p'}{p}\right) - g\left(\frac{p'}{p} - \frac{\rho'}{\rho}\right) \tag{9}$$

$$\frac{B'}{B} = -\frac{\beta}{2}\frac{p'}{p} \tag{10}$$

$$\frac{\partial}{\partial t}\left(\frac{p'}{p}\right) - \Gamma_1\frac{\partial}{\partial t}\left(\frac{\rho'}{\rho}\right) + \left[\frac{d\ln p}{dz} - \Gamma_1\frac{d\ln \rho}{dz}\right]\frac{\partial \xi}{\partial t} = -\frac{\chi_T}{\rho c_v T}\nabla.\mathbf{F}', \tag{11}$$

where ξ denotes the vertical displacement and H is the pressure scale height. The perturbation of equation (3) can be done in a straightforward manner:

$$\nabla.\mathbf{F}_{\mathbf{R}}' = \frac{dF_R'}{dz} = 4\pi\kappa_a\rho(S' - J') + \nabla.\mathbf{F}_{\mathbf{R}}\left(\frac{\kappa'}{\kappa} + \frac{\rho'}{\rho}\right) \tag{12}$$

. The perturbation in the mean intensity J' is determined by perturbing and linearizing the transfer equation (4) which in the first order moment form is

$$\frac{d}{dz}\left(\frac{J'}{J}\right) = \frac{dlnJ}{dz}\frac{\eta'}{\eta} - \frac{dlnJ}{dz}\frac{J'}{J} + \frac{dlnJ}{dz}\frac{\mathcal{H}'}{\mathcal{H}} \tag{13}$$

$$\frac{d}{dz}\left(\frac{\mathcal{H}'}{\mathcal{H}}\right) = \left(\frac{\Delta_t}{2\epsilon H_p} - \frac{dln\mathcal{H}}{dz}\right)\frac{\eta'}{\eta} + \frac{\Delta_t}{2\epsilon H_p}\frac{a'}{a} - \frac{(1+\nabla_c)(4+3\tau_a^2)}{16\epsilon H_p}\frac{J'}{J} + \frac{1}{qH_p}\frac{T'}{T} - \frac{dln\mathcal{H}}{dz}\frac{\mathcal{H}'}{\mathcal{H}}, \tag{14}$$

where $\mathcal{H} = \mathbf{F}/4$ is the Eddington flux, $\eta = \kappa\rho$, H_p is the pressure scale-height at the bottom, and the various other quantities are as defined below.

$$\nabla_c = \frac{J}{S} - 1 \tag{15}$$

is a measure of departure from radiative equilibrium,

$$\Delta_t = \frac{J - J_e}{S} \tag{16}$$

is the ratio of the excess of the mean intensity inside the tube to the Planck function, ϵ and q are the ratios

$$\epsilon = \frac{\tau_r}{\tau_{th}} \tag{17}$$

and

$$q = \frac{\tau_N}{\tau_{th}}, \tag{18}$$

where

$$\tau_{th} = \frac{\rho c_v T H_p}{\mathcal{H}}, \tag{19}$$

is the radiative relaxation time over the length of one pressure scale-height at the bottom,

$$\tau_r = \frac{\rho c_v a^2}{K}, K = \frac{16\sigma T^3}{3\kappa\rho} \tag{20}$$

is the radiative relaxation time across the tube in the optically thick limit and

$$\tau_N = \frac{c_v T}{4\kappa S} \tag{21}$$

is the radiative relaxation time that one obtains in the optically thin limit (with Newton's law of cooling)(Spiegel, 1957), and

$$\tau_a = \kappa\rho a \tag{22}$$

is the depth dependent optical thickness of the tube. We reduce the perturbation equations to a final set of four equations for the four variables ξ, p'/p, J'/J and $\mathcal{H}'/\mathcal{H}$. The optically thick reduction of the set of equations corresponds to taking the limit τ tending to infinity and replacing the mean intensity by the Planck function. The optically thin case is got in the limit τ_{th} approaching zero.

2.3. Boundary conditions

We use closed mechanical boundary conditions and as thermal conditions we impose that there is no incoming radiation from above at the top boundary and that the perturbations are adiabatic at the bottom boundary.

3. Results and Discussion

The action of radiation that we bring out in this study of convective instability and wave motions of the tube is explained conveniently with the help of the graphs shown in Figs. 1 and 2 where we plot the growth rates and frequencies of the fundamental mode, which is the most unstable, respectively, as a function of the surface tube radius a_0 for various values of the plasma β.

3.1. Convective instability

The onset of the convective instability corresponds to the cusps in the curves of fig.1 where the overstable mode's frequency becomes zero and the growth rate shoots up sharply. Comparison with the results obtained with only the lateral exchange in the diffusion approximation (Venkatakrishnan 1986, Hasan 1986) or in the Newton's law of cooling reveal that these earlier treatments overestimate the degree of instability: the growth rates obtained with the present more accurate treatment of radiation with the inclusion of vertical exchange of

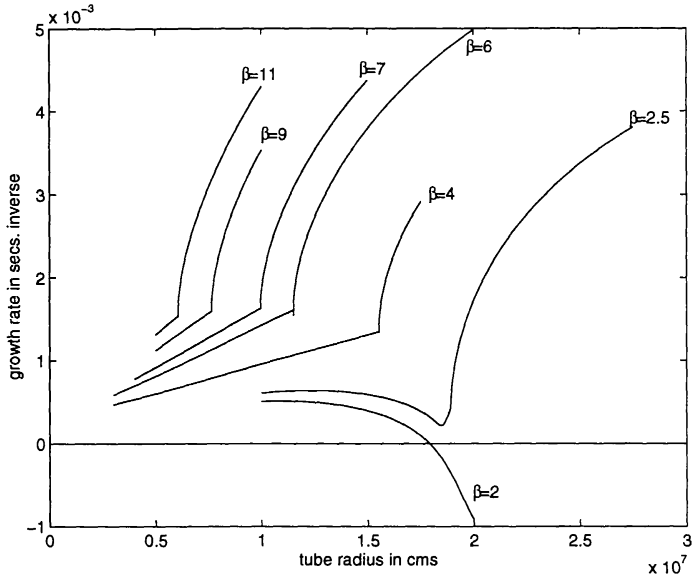

Figure 1. Growth rates of the fundamental mode as a function of the tube radius a_0 at the surface $z = 0$ $(\tau = 1)$ for various values of the plasma β.

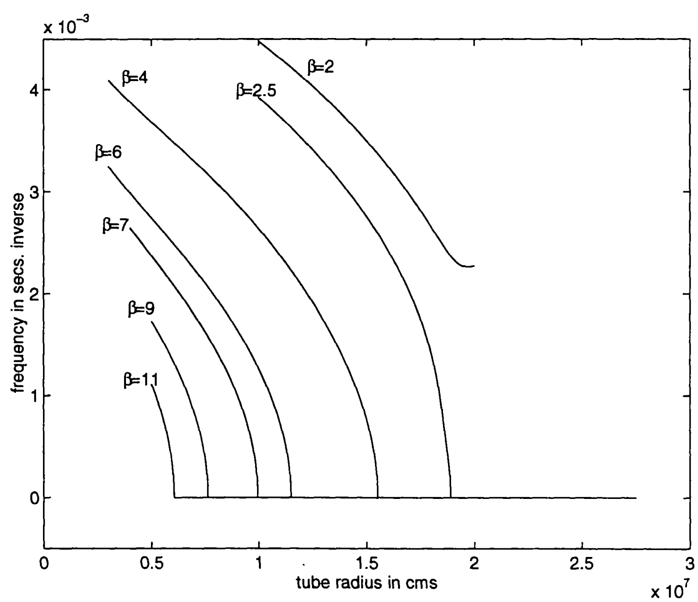

Figure 2. The frequencies of the fundamental mode as a function of the tube radius a_0 at the surface $z = 0$ $(\tau = 1)$ for various values of the plasma β.

radiation are appreciably smaller. Moreover, for a given value of β, i.e., for a tube of given strength, the onset of convective instability requires the size of the tube be greater in the present case than is required when the diffusion approximation is used.

The convective instability is completely suppressed for tubes with the plasma β smaller than 2.45 whatever be its size; this corresponds to a field strength of about 1160 Gauss at $\tau = 1$ inside the tube. This has to be compared with the value of 1350G ($\beta = 1.83$) that Spruit and Zweibel obtained in the adiabatic case. We point out here that the field strength of 1160G that we obtain here does not necessarily imply that all collapsing tubes of weaker fields will attain this unique value and become stable; this value represents a necessary strength for stability against convective collapse and thus can be considered as a minimum strength for stability. A collapsing weaker tube of sufficient size can of course attain an equilibrium collapsed state of field strength higher than this value (cf. Spruit 1979).

3.2. Size-strength distribution for the solar tubes

From the positions of the cusps in the curves of Fig. 1, i.e. from the positions that mark the onset of the convective instability, we pick up the values of the plasma β and the radius a_0; the resulting radius-field strength dependence is shown in Fig. 3, and the corresponding flux-strength distribution in Fig. 4. Comparison of this curve with those observationally produced(Solanki et al. 1996, Lin 1995) shows a remarkable agreement leading to the conclusion that indeed the convective collapse is the cause of the formation of the flux elements on the Sun's surface. Our refined, realistic, and exact treatment of the convective collapse process on the Sun reinforces the conclusions drawn from earlier simplified treatments (Venkatakrishnan 1986, Hasan 1986) and verifies the original suggestion and the physical explanation by Parker (1978) for the concentrated small scale magnetic structures on the Sun.

3.3. Overstability

The characteristics of the overstable mode are explained with the help of Fig. 1 again. The growth rates in the present Eddington approximation are lower than those obtained in the earlier treatments where the vertical losses are not taken care of and use either the diffusion approximation or the Newton's law of cooling. The differences thus demonstrate that overstability is hindered by the vertical losses

We point out that while there is no damping out of oscillations when only lateral exchange takes place and the oscillations' growth rate only asymptotically goes to zero in the limit of large radii, i.e., in the adiabatic limit, the inclusion of vertical radiative losses make the oscillations damp out for radii greater than a particular finite value which is determined by the β. Thus it is clarified that the horizontal exchange between vertically oscillating fluid elements acts to amplify the oscillations while the vertical losses always try to smooth out the fluctuations thereby introducing damping. This *radiative damping* is quite severe for the overstable mode of an intense flux tube on the Sun that it gets completely damped out for tubes of radii larger than a certain critical value.

Finally we note that a tube which can undergo convective collapse for radii greater than a critical value for a given field strength remains overstable for all smaller radii that it can take.

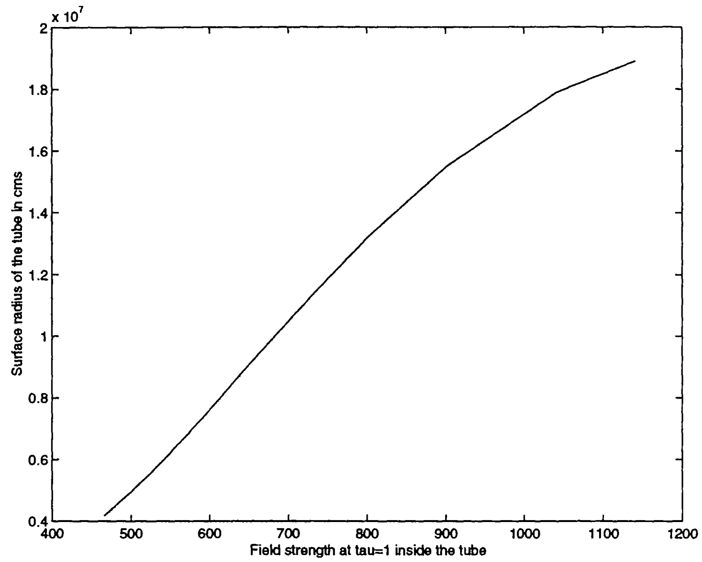

Figure 3. Size-strength relation for the stable tubes; see the text for details

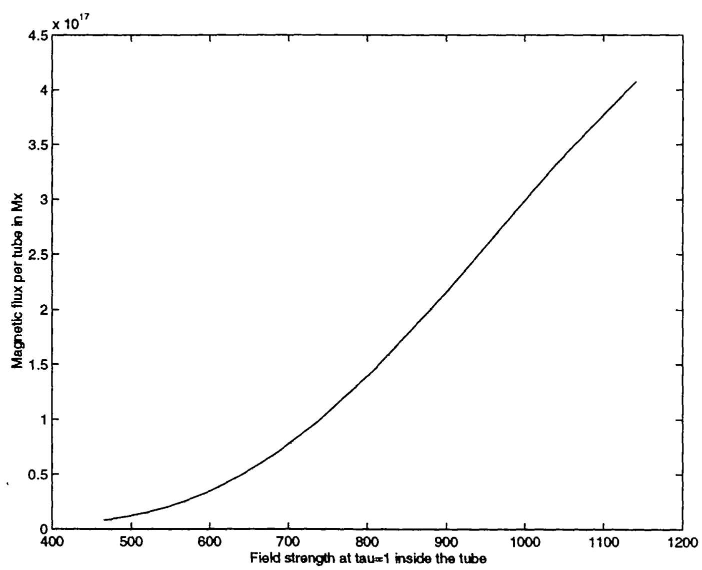

Figure 4. The magnetic flux-strength relation corresponding to the size-strength relation of the previous graph

4. Conclusions

- We have demonstrated that radiative transport has a marked effect on the size-field strength relation for solar flux tubes. Our results can be be applied to solar flux tubes more reliably in view of the more refined treatment of radiative transfer.

- We have generalized the necessary condition for the onset of convective instability in the presence of radiative energy exchange. We find that radiation has a stabilizing influence which is greater for tubes with small radius.
- Overstability of the longitudinal slow mode is shown to be dependent on the tube radius: there is a critical tube radius above which a strong field convectively stable tube's oscillations get damped as a result of vertical radiative losses.

References

Cox, J.P., 1980, Theory of Stellar Pulsation, Princeton University Press.
Hasan, S.S., 1984, ApJ, 285, 851
Hasan, S.S., 1984, A&A, 143, 39
Hasan, S.S., 1986, MNRAS, 219, 357
Hasan, S.S., Kneer, F., & Kalkofen, W. 1998, A&A, 332, 1064
Keller, C.U., et al., 1994, A&A, 286, 626
Kurucz, R.L., 1993, CD-Rom 13
Lin, H., 1995, ApJ, 446, 421
Parker, E.N., 1978, ApJ, 211, 368
Roberts, B. & Webb, A.R., 1978, Sol. Phys. 56, 5
Rogers, F.J., & Iglesias, C. 1992, ApJS, 79, 507
Schussler, M., 1991 in The Sun: A Laboratory for Astrophysics, 191-220, ed. by J.T.Schmelz and J.C.Brown
Solanki, S.K., et al, 1996, A&A, 310, L33-L36
Spiegel, E.A., 1957, ApJ, 126, 202
Spruit, H.C., 1977, Sol. Phys. 55, 3
Spruit, H.C., 1979, Sol. Phys., 61, 363
Spruit, H.C., & Zweibel, E., 1979, Sol. Phys., 62, 15
Steiner, O., Grossman-Döerth, U., Schussler, M., & Knölker, M., 1996, Sol. Phys. 164, 223
Stenflo, J.O., & Harvey, J., 1985, Sol. Phys., 95, 99
Unno, W., & Spiegel, E.A., 1966, PASJ, 18, 85
Venkatakrishnan, P., 1986, Nature, 322, 156
Vernazza, J., Avrett, E., & Loeser, R., 1981, ApJSuppl. 45, 635
Webb, A.R., & Roberts, B., 1978, Sol. Phys., 59, 249
Zayer, I., Solanki, S.K., & Stenflo, J.O., 1989, A&A, 211, 463
Zayer, I., Solanki, S.K., Stenflo, J.O.,& Keller, C.U., 1990, A&A, 239, 356