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Effects of velocity laws on line formation

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Abstract. We calculated the spectral line profiles in a spherically symmetric expanding medium. Complete redistribution of photons in the line for a two level atom is assumed with Doppler profile function in a purely scattering medium. Using transfer equation we have calculated the source functions in the comoving frame and this is employed to simulate the line profiles observed at infinity. We have characterized the envelope of the star by three functions namely (1) Geometrical extension of the medium, (2) Optical depth law, (3) and Velocity law. For various optical depths and velocities we have considered the different ratios of outer to inner radii of the atmosphere to study the effect of sphericity on line formation. We describe the profiles in terms of fluxes integrated over the whole disc versus the normalized frequency. We obtained several P-Cygni type profiles.

Key words : radiative transfer—comoving frame

1. Introduction

It is well known that gases in the outer layers of many O-B supergiants, Wolf-Rayet stars, P Cygni type stars, show evidence of radial expansion. Simulation of spectral lines formed in such media is a matter of some difficulty. This is because of the fact that the photons are redistributed after each absorption and emission. In a medium which is in motion, the line photons can be redistributed in the interval $\nu(1 - V_{max}/c)$ to $\nu(1 + V_{max}/c)$ where ν is the frequency of the photons at the line centre, V_{max} the gas velocity and c the velocity of light. There were several attempts to simulate lines in expanding media in recent years (Hummer & Kunasz 1974; Peraiah 1978, Peraiah & Wehrse 1978; Wehrse & Peraiah 1979). But these calculations were done in the rest frame of the star and therefore were good only up to a maximum of 2 to 3 mean thermal units (mtu). If one wants to calculate the radiation field in the rest frame we have to extend the frequency mesh to the interval of $\nu(1 \pm V_{max}/c)$. This is quite unmanageable because V_{max} is nearly 100 mtu in some stars. This difficulty is avoided in a comoving frame in which the observer need not take into account, the changes in the absorption coefficient due to the continuous Doppler shift. We have considered various types of velocity laws to investigate their effects on line formation for various geometric extensions of the medium. The computational procedure and the results are described below.

2. A brief description of the method in comoving frame

In spherical geometry the terms that should be included in the transfer equation when it is transformed into comoving frame (Chandrasekhar 1945; Mihalas *et al.* 1975; Mihalas 1978; Peraiah 1984) are

$$\left\{(1-\mu^2)\frac{V(r)}{r} + \mu^2 \frac{dV(r)}{dr}\right\}\frac{\partial I(r,\mu,x)}{\partial x}. \qquad \ldots (1)$$

Here $I(r, \mu, x)$ is the specific intensity of the ray with standard frequency x, [$x = (\nu - \nu_0)/\Delta$, Δ being mean thermal Doppler unit, ν_0 and ν are the frequencies at line centre and at any point in the line respectively] making an angle $\cos^{-1}\mu$ at the point r. $V(r)$ is the radial velocity of the gas at r expressed in mean thermal Doppler units. The transfer equation for inward and outward going rays is

$$\pm\mu\frac{\partial I(r,\mu,x)}{\partial r} \pm (1-\mu^2)\frac{\partial I(r,\mu,x)}{\partial \mu}$$
$$= K_L(r)\,[\phi(x)+\beta]\,[S(r,x) - I(r,\pm\mu,x)]$$
$$+ \left\{(1-\mu^2)\frac{V(r)}{r} + \mu^2\frac{dV(r)}{dr}\right\}\frac{\partial I(r,\pm\mu,x)}{\partial x} \qquad \ldots (2)$$

where $K_L(r)$ is the absorption coefficient at the line centre, $\phi(x)$ the Doppler profile function and $S(r, x)$ the source function. The source function is given by

$$S(r,x) = \frac{\phi(x)}{\phi(x)+\beta}S_L(r) + \frac{\beta}{\phi(x)+\beta}S_C(r), \qquad \ldots (3)$$

where

$$S_L(r) = \frac{1}{2}(1-\varepsilon)\int_{-\infty}^{+\infty}\phi(x)\int_{-1}^{+1} I(r,\mu',x)\,dx\,d\mu' + \varepsilon B(T(r)),$$
$$S_C(r) = B(T(r)), \qquad \ldots (4)$$

and ε is the probability per each scattering that a photon will be destroyed by collisional de-excitation, β is the ratio of continuum to line opacity, and $B(T(r))$ is the Planck's function.

We have adopted the *cel* method described by Peraiah (1984) for solving the equations. This is done by suitable descretization in frequency, angle and radius.

Using the sources functions that are obtained in the comoving frame, one can obtain the radiation field observed at infinity by the following procedure : The medium is divided into several shells. The radial velocity V_{rad} at each point on the shell are projected on to the line of sight of the observer. The resultant frequency given by $X = x' + \mu V_{rad}$ where $x' = (\nu' - \nu_0)/\Delta$ and V_{rad} are measured in units of the mean thermal velocity of gas. The optical depth has been calculated in each of these shells along the line of sight. We calculate the transfer of rays along the line of sight. These are parallel rays cutting across the shell boundaries. The specific intensity at each shell boundary becomes the incident radiation in the next shell.

No incident radiation is given at the outer most boundary. The emergent intensity is calculated by the formal solution of the radiative transfer equation.

$$I(\tau_L) = I_0 \exp[-\tau_L] + \int_0^{\tau_L} S(t) \exp[-(\tau_L - t)]dt \qquad \dots (5)$$

where $S(t)$ is the source function. We estimate the specific intensity $I(\tau_L)$ at the boundary of each shell. Finally we calculate the flux by the integral

$$F(x) = 2\pi \int_A^B I(p, x) p dp \qquad \dots (6)$$

where p is the perpendicular distance from the centre of the star to the ray along the line of sight.

3. Computational procedure and results

We have chosen trapezoidal points for frequency integration. We have employed 9 frequency points ($I = 9$) and 2 angles ($m = 2$). Since B and A represents the outer and inner radius of the spherical shell respectively, we have taken $B/A = 2, 5, 10, 100$. The velocities are measured in Doppler units. $V = 1, 5, 10, 25$ in mean thermal units. The velocity law which we have taken is (a) the exponential velocity law $V(r) = V_0 \exp(-((r - r_0)/r_0))$, (b) the power law $V(r) = V_0(r_0/r)^n$ where $n = 1, 2, 3, 5$, and r_0 is inner radius of the star. The optical depth law is $\tau(r) = \tau_0 \exp(-((r - r_0)/r_0))$. We describe the profiles in terms of fluxes integrated over the whole disc versus the normalized frequency.

We define another quantity $Q = X_Q/X_{max}$; $-1 \le Q \le 1$. The purpose of this kind of representation is to accommodate all profiles for different velocities in a single figure. We plotted F_Q/F_C where F_Q and F_C are the integrated fluxes at frequency X_Q and in the continuum respectively.

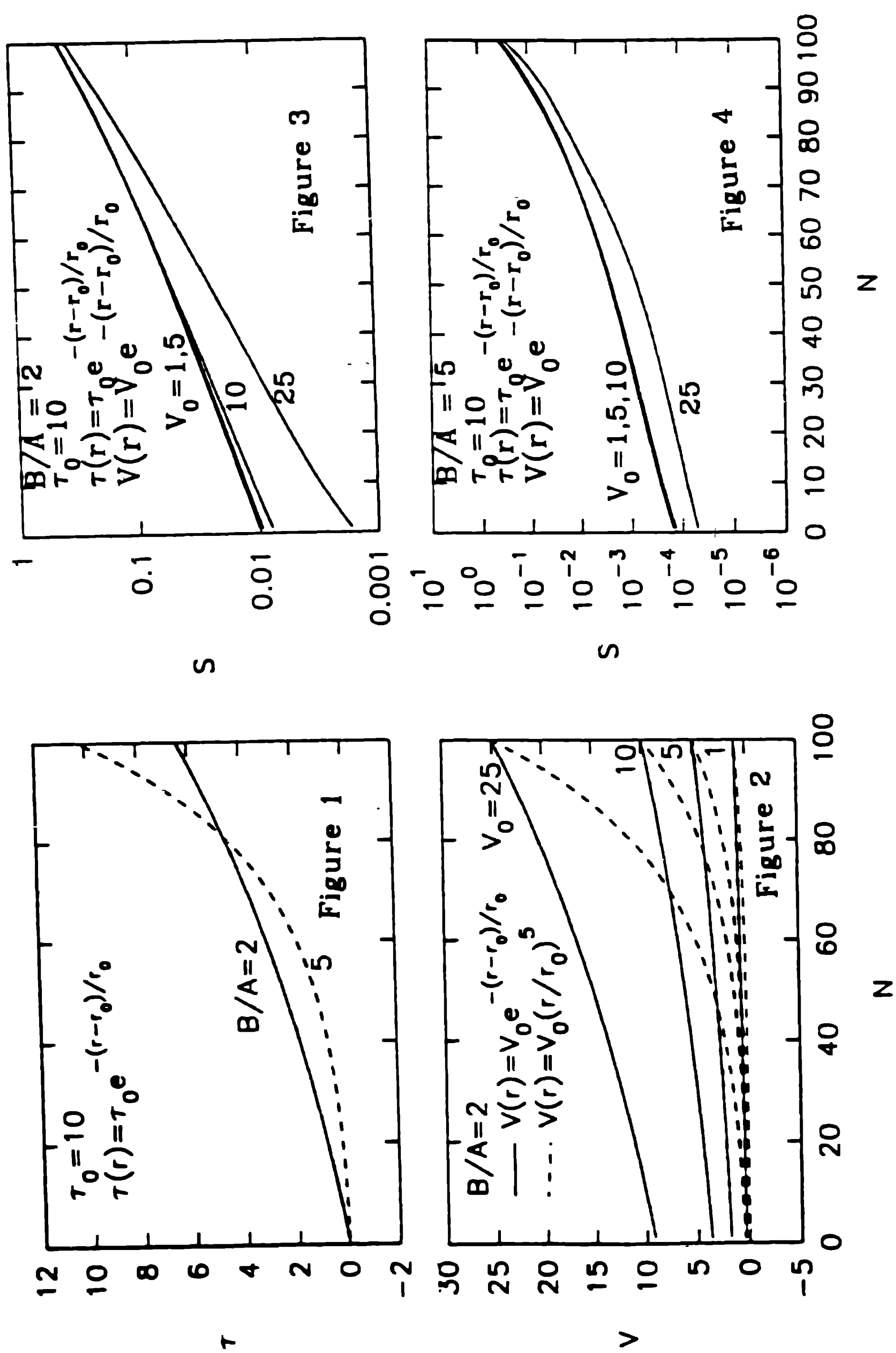
In figure 1, we have plotted the optical depth distribution over the shells for B/A =2 and 5. The velocity distribution due to two different velocity laws are plotted in figure 2, for various initial velocities V_0. In figure 3, we have shown the source functions for $B/A = 2$ and $\tau_0 = 10$. We see that the source function is the same throughout the atmosphere for $V_0 = 1$, 5, 10. But for $V_0 = 25$, the source function is reduced due to easy escape of photons from the outer layers. Figure 4, is the same as figure 3, but for $B/A = 5$. In figure 5, we have plotted the flux profiles versus Q for $B/A = 2$, $\tau_0 = 10$ and exponential velocity law. In the bottom half of the figure, for the same parameters, we have plotted the profiles for $V(r) = V_0(r_0/r)^5$. In figure 6, we have plotted the flux profiles for the same parameters as in figure 5, but for $B/A = 5$. In the static case we get the symmetric profile. We obtain blue shifted absorption and red shifted emission when the medium is expanding. We get deeper absorption core for power-law for velocity as compared to the flux profiles for the exponential velocity law. We obtained similar results for $B/A = 5$ and $V(r) = V_0(r_0/r)^5$.

Figure 3

Figure 4

Figure 1

Figure 2



Figures 1-4.

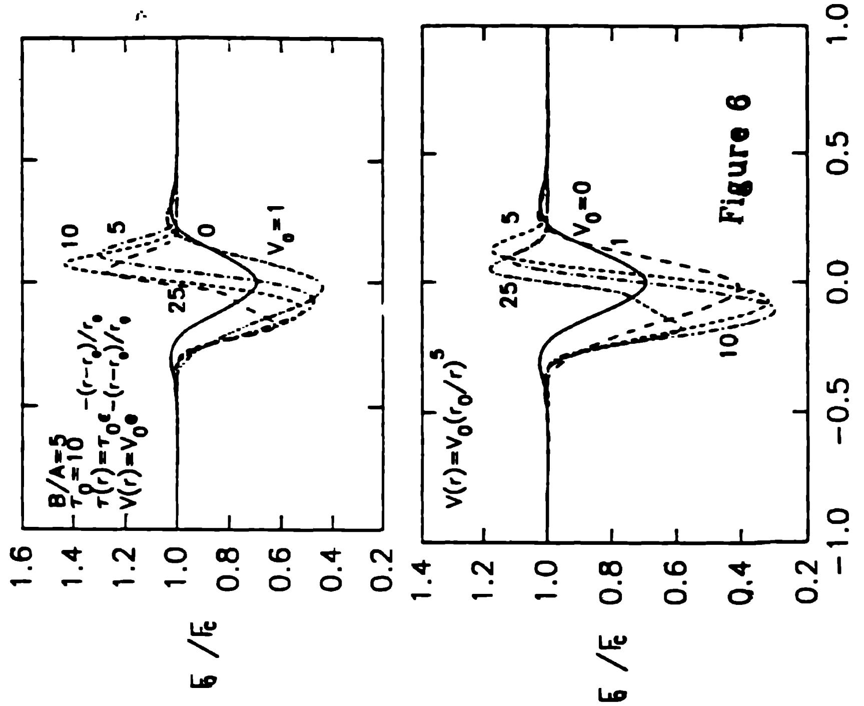


Figure 6.

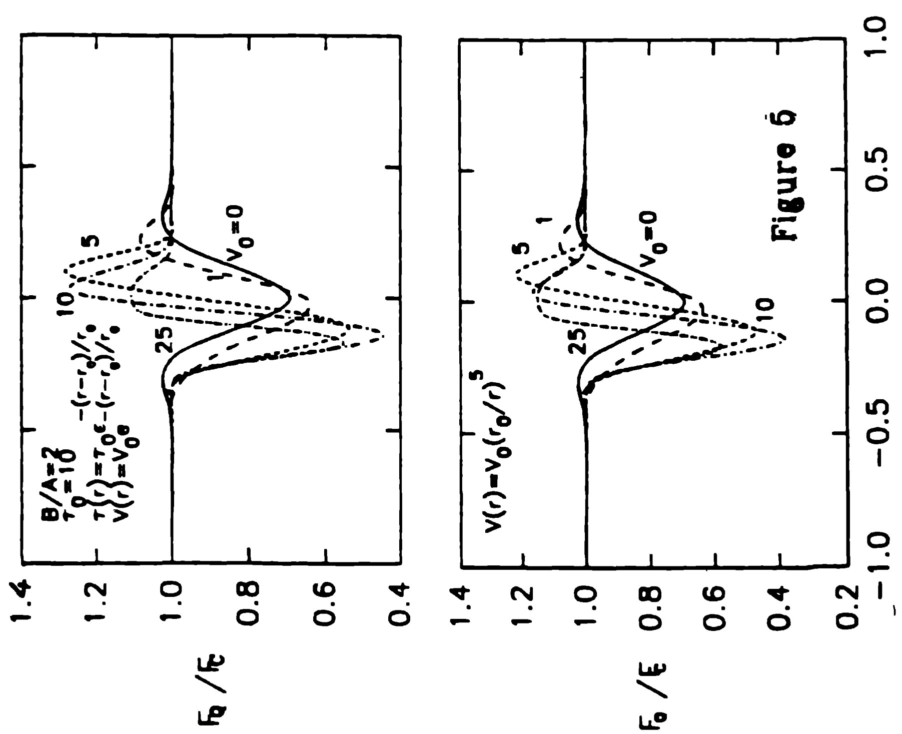


Figure 5.

4. Conclusions

The line profiles has been calculated for different velocity laws. We get deep absorption core for power law for velocity compared to the exponential velocity law. The profiles were calculated for different optical depths and for various geometrical extension of the medium.

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