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Faraday rotation versus stimulated Raman scattering in intense sources

R. T. Gangadhara and V. Krishan
Indian Institute of Astrophysics, Bangalore 560 034

Abstract. A coherent plasma process such as the stimulated Raman scattering plays significant role in bringing about the changes in polarization characteristics of radiation. It is found that some of the observed polarization properties such as rapid temporal variations, sense reversal, rotation of the plane of polarization and change of nature of polarization in the case of pulsars and quasars, could be accounted for through stimulated Raman scattering.

Key words : polarization—Raman scattering—plasma—variability—pulsars—quasars

1. Introduction

The change in polarization of an electromagnetic (EM) wave due to its propagation in a magnetized plasma as well as due to an electron scattering is well known. The Faraday rotation is recognized to be the most common cause of the rotation of the plane of polarization of an EM wave. In a plasma, the spectral components of radiation of finite band-width travel different path lengths. This may lead to depolarization. Any change in the direction of the magnetic field also manifests itself through polarization variation. The lack of polarization in quasars and Seyfert galaxies could be due to depolarization effects.

In pulsars, the ambient magnetic field determines the orientation of the polarization ellipse, but with an ambiguity of $\pi/2$. The relative frequency of occurrence of the modes is a strong function of both pulse longitude and pulsar. The modes are actually elliptically polarized such that one angle correlates with one sense of circular polarization, the orthogonal angle correlates with the other sense. Several pulsars exhibit one or more reversals of the sense of polarization through the profile. Three possible reasons for low polarization, apart from the Faraday rotation, electron scattering and magnetic field orientations, are that (Manchester & Taylor 1977) the subpulses at a given longitude may (1) be themselves weakly polarized; (2) be divisible into groups with orthogonal polarization; or (3) have randomly varying position angles and sense of circular polarization.

In this paper, it is shown that in strong radio sources, such as quasars and pulsars, the rapid temporal variations, sense reversal of rotation of electric field, rotation of plane of polarization and change of nature of polarization may be accounted for through stimulated

Raman scattering (SRS). The physics of SRS in a plasma has been explained in many papers and books (e.g. Liu & Kaw 1976; Kruer 1988). The polarization changes through SRS may take place in accretion disks, the emission line regions and the intercloud medium of active galactic nuclei and also in the emission region of pulsars. We make a comparison between SRS and the well known process of the Faraday rotation and investigate the conditions under which growth rate SRS dominates over the Faraday rotation.

2. Stimulated Raman scattering

We begin with a model consisting of a pulsar with nonthermal component of radiation interacting with the plasma in the emission region at a distance $r = 100R_{NS} = 10^8$ cm (Neutron star radius $R_{NS} \approx 10$ km). In the case of a quasar, we consider a black hole surrounded by a plasma which extends to a few parsecs. The nonthermal continuum is considered as a pump which drives SRS.

Consider a large amplitude elliptically polarized EM wave,

$$\vec{E}_1 = \varepsilon_1[\cos(\vec{k}_1 \cdot \vec{r} - \omega_1 t)\hat{e}_1 + \alpha_1 \cos(\vec{k}_1 \cdot \vec{r} - \omega_1 t + \delta_1)\hat{e}_2], \quad \dots (1)$$

propagating in a plasma of density n_0 and temperature T_e.

We can think of $\vec{E}_1$ as the superposition of two linearly polarized waves : $\vec{E}_{11} = \varepsilon_1 \cos(\vec{k}_1 \cdot \vec{r} - \omega_1 t)\hat{e}_1$ and $\vec{E}_{12} = \alpha_1 \varepsilon_1 \cos(\vec{k}_1 \cdot \vec{r} - \omega_1 t + \delta_1)\hat{e}_2$. Let

$$\delta n_{e1} = \delta n_1 \cos(\vec{k} \cdot \vec{r} - \omega t) \quad \text{and} \quad \delta n_{e2} = \delta n_2 \cos(\vec{k} \cdot \vec{r} - \omega t + \delta_e), \quad \dots (2)$$

be the density perturbations coupled to the electric fields $\vec{E}_{11}$ and $\vec{E}_{12}$, respectively, in a plasma with equilibrium density n_0.

The electric fields $\vec{E}_\pm$ of the EM wave scattered through the angles $\phi_\pm$ with respect to $\vec{k}_1$ are given by

$$\vec{E}_\pm = \varepsilon_s [\cos(\vec{k}_\pm \cdot \vec{r} - \omega_\pm t)\{\cos(\phi_\pm)\hat{e}_1 - \sin(\phi_\pm)\hat{e}_3\} + \alpha_\pm \cos(\vec{k}_\pm \cdot \vec{r} - \omega_\pm t + \delta_\pm)\hat{e}_2]. \quad \dots (3)$$

The wave equation for the scattered EM wave is given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}_\pm = \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t}, \quad \dots (4)$$

where c is the velocity of light. Here $\vec{J}$ is the current density and its components are

$$J_1 = -en_{e1}u_{e1} = -e(n_0 + \delta n_{e1})u_{e1}, \quad J_2 = -en_{e2}u_{e2} = -e(n_0 + \delta n_{e2})u_{e2},$$

$$J_3 = -en_0 u_{e3}, \quad \dots (5)$$

where u_{e1}, u_{e2} and u_{e3} are the components of the oscillation velocity $\vec{u}_e$ of electrons in the radiation fields $\vec{E}_1$ and $\vec{E}_\pm$.

Now from the components of equation (4) we obtain

$$D_\pm \varepsilon_\pm \cos(\phi_\pm) = -\frac{4\pi e^2}{m_0}\varepsilon_1 \delta n_1, \quad \dots (6)$$

$$D_\pm \alpha_\pm \varepsilon_\pm \cos[\vec{k}\cdot\vec{r} - \omega t \pm (\delta_\pm - \delta_\text{i})] = -\frac{4\pi e^2}{m_0}\alpha_\text{i}\varepsilon_\text{i}\delta n_2 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta_\text{e}), \ldots \quad (7)$$

where $D_\pm = k_\pm^2 c^2 - \omega_\pm^2 + \omega_{\text{pe}}^2$ are the dispersion relations for the stokes mode $(\vec{k}_-, \omega_-)$ and the anti-stokes mode $(\vec{k}_+, \omega_+)$, when the following resonant conditions are satisfied :

$$\omega_\text{i} - \omega = \omega_-, \quad \omega_\text{i} + \omega = \omega_+, \quad \vec{k}_\text{i} - \vec{k} = \vec{k}_-, \quad \vec{k}_\text{i} + \vec{k} = \vec{k}_+. \quad \ldots (8)$$

Similar to the conditions (8), equation (7) gives the conditions between the phases $\delta_\pm = \delta_\text{i} \pm \delta_\text{e}$. Dividing the equation (7) by equation (6), we have $\alpha_\pm = \alpha_\text{i} R \cos(\phi_\pm)$, where $R = \delta n_2/\delta n_1$. In the linear theory, it is not possible to determine R, but one expects that it may not differ too much from the value of α_i.

Now, we determine the electron density perturbations δn_1 and δn_2, using the Vlasov equation for the low frequency response of electrons, and obtain

$$\left(1 + \frac{1}{\chi_\text{e}}\right)(1 - R)\delta n_1 = \frac{k^2}{8\pi m_0 \omega_\text{i}^2} A, \quad \ldots (9)$$

where $A = (\alpha_\text{i}\alpha_- - \cos\phi_-)\varepsilon_\text{i}\varepsilon_- + (\alpha_\text{i}\alpha_+ - \cos\phi_+)\varepsilon_\text{i}\varepsilon_+$.

From equations (6), (7) and (9), we obtain

$$\left(1 + \frac{1}{\chi_\text{e}}\right)(1 - R) = \frac{v_0^2 k^2}{2}\frac{(1 - R\alpha_\text{i}^2)}{(1 + \alpha_\text{i}^2)}\left(\frac{1}{D_-} + \frac{1}{D_+}\right), \quad \ldots (10)$$

where $v_0 = e\varepsilon_\text{i}\sqrt{1 + \alpha_\text{i}^2}/m_0\omega_\text{i}$ is the quiver velocity of electrons in the field of incident EM wave. Its relation with the luminosity of the source is given by

$$v_0 = \frac{e}{m_0}\left(\frac{2L}{r^2 c}\right)^{1/2}\frac{1}{\omega_\text{i}}. \quad \ldots (11)$$

Equation (10) is the plasma dispersion relation describing the SRS of EM wave.

3. Stokes parameters

We use stokes parameters to study the polarization properties of the incident and scattered EM waves. For the incident wave : $I_\text{i} = c(1 + \alpha_\text{i}^2)\varepsilon_\text{i}^2/8\pi$, $Q_\text{i} = (1 - \alpha_\text{i}^2)I_\text{i}/(1 + \alpha_\text{i}^2)$, $U_\text{i} = 2\alpha_\text{i} I_\text{i}\cos(\delta_\text{i})/(1 + \alpha_\text{i}^2)$ and $V_\text{i} = -2\alpha_\text{i} I_\text{i}\sin(\delta_\text{i})/(1 + \alpha_\text{i}^2)$. The sense of rotation of the electric field is given by $\sin(2\beta_\text{i}) = V_\text{i}/I_\text{i}$. The magnitudes of the principle axes of the ellipse are $a_\text{i} = I_\text{i}|\cos(\beta_\text{i})|$ and $b_\text{i} = I_\text{i}|\sin(\beta_\text{i})|$. The orientation of the major axis of the ellipse relative to $\hat{e}_1$ axis, is given by $\tan(2\chi_\text{i}) = U_\text{i}/Q_\text{i}$.

Using the conditions $\delta_\text{e} = \pi$, $\delta_- = \delta_\text{i} - \pi$ and $\alpha_- = \alpha_\text{i} R$, we find the stokes parameters for the scattered wave : $I_- = c(1 + \alpha_\text{i}^2 R^2)\varepsilon_\text{s}^2/8\pi = c\varepsilon_-^2/8\pi$, $Q_- = (1 - \alpha_\text{i}^2 R^2 \cos^2\phi_-) \times$

$I_-/(1+\alpha_1^2R^2\cos^2\phi_-)$, $U_- = -2\alpha_1 R\cos\phi_- I_-\cos(\delta_1)/(1+\alpha_1^2R^2\cos^2\phi_-)$ and $V_- = 2\alpha_1 R \times \cos\phi_- I_-\sin(\delta_1)/(1+\alpha_1^2R^2\cos^2\phi_-)$. The sense of rotation of the electric field is given by $\sin(2\beta_-) = V_-/I_-$. The magnitudes of the principle axes of the ellipse are $a_- = I_-\,|\cos(\beta_-)|$ and $b_- = I_-\,|\sin(\beta_-)|$. The orientation of the major axis of the ellipse relative to $\hat{e}_1$ axis, is given by $\tan(2\chi_-) = U_-/Q_-$.

Using $\omega = \omega_e + i\Gamma$, we numerically solve equation (10) including all the damping effects, where ω_e is the electron plasma wave frequency and Γ is the growth rate. The typical values of the plasma and radiation parameters at a distance $r = 100R_{NS} = 10^8$ cm, (Neutron star radius $R_{NS} \approx 10$ km), in a pulsar are : electron density $n_e = n_8 \times 10^8$ cm^{-3}, temperature $T_e = T_5 \times 10^5$ K and luminosity in the band $\Delta\nu < \nu = 600$ MHz is $L = L_{30} \times 10^{30}$ erg/sec.

We know from the observations of pulsar PSR 1133 + 16 $I_1 = 10^{-20}$ erg cm^{-2} sec^{-1} Hz^{-1} at the radio frequency $\nu_1 = 600$ MHz. The Manley-Rowe relation gives the relation between I_0 and I_-, given by $I_- = (1-\omega/\omega_1)I_1$. For $\omega \approx \omega_{pe}$ and $\omega_1 = 2.5\omega_{pe}$, we get $I_- = 0.6I_i$.

In table 1, the values of stokes parameters for the incident and the forward scattered EM waves in a pulsar PSR 1133 + 16 are listed. It shows that a circularly polarized incident wave with counter-clockwise sense ($\delta_1 = \pi/2$, $\alpha_1 = 1$) scatters into (i) linearly polarized wave when $R = 0$, (ii) an elliptically polarized waves with clockwise sense when $R = 0.4$ and 0.8, (iii) circularly polarized wave with clockwise sense when $R = 1$.

4. Faraday rotation versus stimulated Raman scattering

A linearly polarized EM wave which is incident on a plasma, will be Faraday rotated through Ω_F rad, where Ω_F is given by (Lang 1974)

$$\Omega_F = \frac{2.36\times10^4}{\nu^2}\int_0^L n_e H\cos(\theta)\,dl \quad \text{rad}, \qquad \ldots (12)$$

after traversing a thickness, L, of the plasma. Here, H is the magnetic field and θ is the angle between the line of sight and the direction of the magnetic field. For $n_e = n_9 \times 10^9$ cm^{-2}, $H = H_{-3} \times 10^{-3}$ G and $\nu = \nu_9 = 10^9$ Hz in the broad-line region of quasar, we get $\Omega_F = 2.36 \times 10^{-8}\, n_9 H_3 L/\nu_9^2$ rad. From the equation (10), we find $\log[t_e] = -5$ sec and $\chi_- = 0.65$ rad for $\alpha_1 = 0.75$ and $\chi_i = -0.644$ rad. The angle through which plane of polarization rotated due to SRS is given by $\Omega_{SRS} = \chi_- - \chi_1 = 1.294$ rad during the time $t_e = 10^{-5}$ sec. To compare Ω_F and Ω_{SRS} we need to convert t_e into the light travel distance $l = ct_e = 3 \times 10^5$ cm. Now, for $n_9 = 5$, $H_{-3} = 1$, and $\nu_9 = 6.366$ we get $\Omega_F = 8.735 \times 10^{-4}$ rad much smaller than Ω_{SRS}.

5. Conclusion

Similar to frequency and wave number matching conditions (see equation (8)) we found conditions between the phases δ_1, $\delta_\pm$ and δ_e in the process of three-wave interaction. Through SRS the clockwise polarized radiation can change into counter-clockwise polarized radiation and vice versa. In addition, circularly polarized wave can change into a linearly polarized, a circularly polarized or an elliptically polarized wave or vice versa, depending upon the value of R. The plane of polarization gets rotated through an angle $(\chi_- - \chi_1)$. Compared to the Faraday rotation the SRS is a faster process.

Table 1. Stokes parameters of the incident and forward scattered EM waves for PSR 1133 + 16

Parameters (erg $cm^{-2}sec^{-1}Hz^{-1}$)	Incident wave $\delta_1 = \pi$, $\alpha_1 = 0.5$	Scattered wave $\delta_- = 0$			
		$R = 0$	$R = 0.4$	$R = 0.8$	$R = 1.2$
I	1.0000E–20	6.0000E–21	6.0000E–21	6.0000E–21	6.0000E–21
Q	6.0000E–21	6.0000E–21	5.5385E–21	4.3448E–21	2.8235E–21
U	–8.0000E–21	0.0	2.3077E–21	4.1379E–21	5.2941E–21
V	0.0	0.0	0.0	0.0	0.0
χ (rad)	–0.4636	0.0	0.1974	0.3805	0.5404
a	1.0000E–20	6.0000E–21	6.0000E–21	6.0000E–21	6.0000E–21
b	0.0	0.0	0.0	0.0	0.0
Sense of rotation	...	...	...	...	...
Nature	linear	linear	linear	linear	linear
Parameters (erg $cm^{-2}sec^{-1}Hz^{-1}$)	Incident wave $\delta_1 = 3\pi/4$, $\alpha_1 = 1$	Scattered wave $\delta_- = -\pi/4$			
		$R = 0$	$R = 0.4$	$R = 0.8$	$R = 1.2$
I	1.0000E–20	6.0000E–21	6.0000E–21	6.0000E–21	6.0000E–21
Q	0.0	6.0000E–21	4.3448E–21	1 3171E–21	–1.0820E–21
U	–7.0711E–21	0.0	2.9260E–21	4.1392E–21	4.1731E–21
V	–7.0711E–21	0.0	2.9260E–21	4.1392E–21	4.1731E–21
x (rad)	—	0.0	0.2963	0.6314	–0.6586
a	9.239E–21	6.0000E–21	5.8064E–21	5.5705E–21	5.5618E–21
b	3.8268E–21	0.0	1.5118E–21	2 2291E–21	2.2510E–21
Sense of rotation	counter-clockwise	—	clockwise	clockwise	clockwise
Nature	elliptical	linear	elliptical	elliptical	elliptical
Parameters (erg $cm^{-2}sec^{-1}Hz^{-1}$)	Incident wave $\delta_1 = \pi/2$ $\alpha_1 = 1$	Scattered wave $\delta_- = -\pi/2$			
		$R = 0$	$R = 0.4$	$R = 0 8$	$R = 1.2$
I	1.0000E–20	6.0000E–21	6.0000E–21	6.0000E–21	6.0000E–21
Q	0.0	6.0000E–21	4.3448E–21	1.3171E–21	0.0
U	0.0	0.0	0.0	0.0	0.0
V	–1.0000E–20	0 0	4.1379E–21	5.8536E–21	6.0000E–21
χ (rad)	—	0.0	2.9170E–17	1 3613E–16	—
a	7.0711E–21	6.0000E–21	5.5709E–21	4 6852E–21	4.2426E–21
b	7.0711E–21	0.0	2.2283E–21	3.7482E–21	4.2426E–21
Sense of rotation	counter-clockwise	—	clockwise	clockwise	clockwise
Nature	circular	linear	elliptical	elliptical	circular

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